State-contingent bank-regulation when the states are not well-known*

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Abstract

This paper studies state-contingent policies in the context of bank capital regulation. Unlike a one-size-fits-all policy, state-contingent capital regulation can be aligned to individual banks' riskiness. However, when riskiness is not perfectly observable, or when supervisory reviews like stress tests provide noisy indicators of banks' characteristics, state-contingent policy can lead to excessive (insufficient) regulation of a less (more) risky bank. This can diminish banks' ex-ante incentives to improve their risk profile, and even reduce welfare relative to a state-independent policy. We show that with information frictions, state-contingent policies must have limits that depend on the accuracy of supervisory reviews.
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1 Introduction

State-contingent (SC) policies are more effective in improving state-wise welfare outcomes compared to state-independent (SI) policies. This is because unlike an SI policy, an SC policy can be calibrated according to future contingencies. Yet, SC policies are not always implementable, such as when all future contingencies are not known ex-ante, or when they cannot be fully observed ex-post. In the presence of such information frictions, an SC policy can turn out to be misdirected ex-post and create adverse incentives ex-ante. This can decrease overall welfare, despite the fact that relative to an SI policy, an SC policy improves ex-post welfare.

A compelling application of SC policy can be found in the context of bank capital regulation. Post-crisis reforms, especially Pillar 2 of Basel III, impose capital-ratio buffers or

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capital-planning restrictions based on bank-specific characteristics such as riskiness, liquidity, profitability, and governance. To gain information on these characteristics, regulators use multiple tools, such as supervisory audits, extensive asset quality reviews, and stress tests. The precision of such assessments, however, is not guaranteed. As a result, policy actions are often based on noisy signals of banks' *types*. This poses welfare trade-offs associated with the design of bank-specific capital requirements. In this paper, we develop a parsimonious model of bank capital regulation under information frictions, and study the attendant welfare trade-offs. We proceed in the following two steps.

First, we develop a baseline model with a rationale for bank capital regulation. Banks accept deposits from households and invest in risky assets. Deposits are covered by an insurance scheme, which we assume to be mis-priced.[1] As a result, banks over-borrow, which increases the probability of bank failures. Failures entail a resolution cost that is borne by the deposit insurance scheme. Since the resolution cost is not internalised by the bank (due to mis-pricing), the model exhibits a so-called bank-failure externality. A minimum capital-ratio requirement partially mitigates this externality by restricting bank borrowing.[2] But it also limits the amount of investment in the economy, thereby posing a welfare trade-off. Hence, as capital regulation tightens, welfare first increases as the positive welfare effect of lower borrowing (via lower expected resolution costs) outweighs the negative welfare effect of lower output. However, beyond a certain threshold, further tightening of capital regulation leads to lower welfare. The resulting inverted U-shaped welfare profile underpins the optimal regulation. Finally, we show that as bank resolution costs increase, the gains from a smaller probability of bank default are higher, as a result of which the optimal capital-ratio requirement increases.

Second we extend the baseline model by adding uncertainty about banks' risk-return profile – or *type* – to study the optimal SC policy. In the context of this model, SC policy is defined as a bank-type specific capital requirement. The probability that a bank is of low- or high-type ex-post depends on the effort it exerts ex-ante. As such, effort exerted increases as the wedge between expected charter value of being high- versus low-type increases. SC policy better aligns regulation to banks' types, but in the process, it affects the charter value wedge and alters banks' incentives towards exerting effort.

Information frictions between banks and regulators play a crucial role in the design of SC policies. We consider several cases. First is where bank's type is public information. In this case, we show that the optimal capital regulation is fully state-contingent, i.e. bank-type specific. Next we assume that banks' type is its private information. In this case, capital regulation cannot be state-contingent.

Stress tests allow regulators to obtain a potentially noisy signal about banks' types. Introducing this possibility in our model delivers the following key insight. The extent to which capital requirements should be bank-type specific depends on the accuracy of stress tests in differentiating between high- and low-type banks. On the one end, when stress

[1] A mispriced deposit insurance can be a consequence of an inability of the insurer to observe banks' risk profiles.

[2] Formally, we show that capital regulation can implement the *second best*, i.e. the solution to a constrained planner's problem, but it cannot achieve the *first best*, i.e. the solution to an unconstrained planner's problem.

tests are poorly designed so that the probability that a bank is misclassified is sufficiently high, we show that the optimal policy should not be state-contingent at all. This is because when misclassification is possible, a high-type bank can fail the test and be penalised. This can be very costly as the shadow cost of capital constraints is higher for high-type banks. In turn, misclassification can diminish ex-ante incentives to exert effort towards becoming a high-type. Therefore, imposing any penalty on banks that fail the test is sub-optimal. On the other end, when stress tests are sufficiently accurate, they can help improve welfare ex-post by revealing information about banks' types and enabling regulation that is better aligned to banks' specific characteristics. In this case, we show that penalties for failing the test can elicit greater effort from banks ex-ante and improve welfare.

Designing a more accurate stress test can be costly and challenging. For one, there are resource costs for both banks and regulators. Second, there may be technical limits on how accurately forward looking indicators of banks' risk characteristics can be estimated. These considerations imply that there is a welfare trade-off associated with designing a more accurate stress test. We show that accuracy of the stress test and the specificity of regulation (i.e., size of penalty for failing the test) increase hand-in-hand as the the cost of designing a more accurate test decreases.

2 Related Literature

Post-crisis reforms reflect a broad consensus among policy makers and academics about the need to regulate bank capital (Admati and Hellwig [2014]). Yet, consensus on specific elements of these reforms continues to evolve. A case in point is bank-specific capital requirements based on supervisory reviews such as stress tests. This paper explores some of the fundamental trade-offs associated with such state-contingent policies.

A large literature provides several rationales for bank capital regulation. Kara and Ozsoy [2016] justify it on the basis of fire-sale externalities, while (Christiano and Ikeda [2016]) cite a moral hazard issue between banks and their creditors as a rationale. Implicit (too-big-to-fail) guarantees (Nguyen [2015]), and household preference for safe and liquid assets (Begenau [2019]) are other reasons why capital regulation is desirable.[3] This paper provides a different rationale. Using a simple model, we show that capital regulation can mitigate socially costly over-borrowing that can result from a mispriced deposit insurance scheme. Our approach is related to that by Merton [1977] and Van den Heuvel [2008] who show that a mispriced deposit insurance induces banks to increase the riskiness of their assets, and to Santos [2001] who shows that over-borrowing can lead to risk-shifting.

Turning to stress tests, most of the literature has focussed on disclosure policy. Goldstein and Leitner [2018] examines the effect of public disclosure of test results on ex-ante risk sharing by banks, while Corona et al. [2017] studies the effect of disclosure policy on co-ordinated risk taking by banks in the presence of bail-outs. Parlasca [2019] studies the time-inconsistency problem associated with disclosures. Orlov et al. [2018] study optimal disclosure policies as a function of the level of aggregate risk in the economy. This paper

[3]We refer the reader to Santos [2001] for a more extensive review of the literature on bank capital regulation.

takes disclosure of stress test results as given, and instead studies how the accuracy of stress test results affects the resulting regulatory action. Our pursuit is broadly related to that of Prescott [2004] who assess the optimal frequency of costly supervisory audits so that banks are incentivised to truthfully report their riskiness. Like in our paper, Prescott [2004] shows that poorly executed audits can create adverse incentives ex-ante.

Our paper relates to studies on state-contingent policies more generally. Relative to a one-size-fits-all policy, state-contingent policies pose several benefits. For instance, Nagarajan and Sealey [1998] show that under contracting frictions, a state-contingent pricing of deposit insurance can achieve the first-best allocation. Likewise, Marshall and Prescott [2001] show that augmenting a uniform ex-ante capital regulation with ex-post state-contingent fines can increase welfare. However, despite these advantages, state-contingent policies may not always be feasible or optimal. For one, Lohmann [1992] shows that when all future states are not known ex-ante, it is sub-optimal to commit to a state-contingent policy. This paper explores a complementary information friction, which is the partial observability or verifiability of future states. Marshall and Prescott [2006] consider such a setup, and show that state-contingent fines can deter bank risk taking and separate low- and high-type banks. A key difference is that the observability of bank-type in their model is taken as given, while in our model the regulator can choose the accuracy with which it observes banks' types. Relatedly, like in our paper, Ahnert et al. [2018] show that sensitivity of regulation to signals about banks' types should depend on its precision. Yet, our specific conclusions differ. They show that starting from a high-degree of signal precision, lower precision implies greater sensitivity of regulation to risk, while our model suggests that such a strategy can decrease welfare by creating adverse incentives ex-ante. This difference stems from the fact that in their paper, banks cannot affect the distribution of the states in which it is regulated.

Our analysis implies that when there is insufficient certainty about banks' risk characteristics, there should be limits on the discretion regulators have in terms of ex-post regulation. As such, our paper speaks to the following trade-off. While rules facilitate the formation of expectations about future policy, they also tie the hands of a regulator. Discretion can help improve outcomes ex-post, but can worsen outcomes ex-ante by creating adverse incentives. A large literature examines this trade-off in the context of monetary (e.g. Kydland and Prescott [1977], Barro and Gordon [1983]) or macro-prudential policy (e.g. Jeanne and Korinek [2013], Bianchi and Mendoza [2015]). Ours is one of the few papers to highlight this perspective in the context of micro-prudential regulation of banks. One exception is Greenwood et al. [2017] who argue that because ex-ante rules can be gamed by banks, regulators must have some discretion. But they also acknowledge that limits to discretion are desirable when all future contingencies are unknown, or when there is a risk that discretion is perceived as lack of transparency.

3 Baseline Model

The model economy lasts two periods: 0 and 1, and consists of a representative household, a bank, a government, and a regulator. The household receives an unconditional income endowment $\bar{Y}$ on each date. It decides how much to consume, c, and how much to deposit, d, in the bank. The deposit is protected against bank failure by an insurance scheme run by the government. As such, deposits are risk-free. They pay a gross return of R on date-1. The household also receives dividends from the bank on date-1, and is taxed lump-sum by the government.[4] Taxes fund the deposit insurance scheme. Since the tax is imposed as a lump-sum, the insurance scheme is mispriced.

The bank has a capital endowment of k on date-0, and issues deposits d to raise funding. It invests $k + d$ in a risky project that pays $\psi g(k + d)$ on date-1, where $g(.)$ is a decreasing returns to scale (DRS) return function and ψ is an investment shock that follows a distribution F and density f. We assume that F is common knowledge. The bank's deposit liabilities on date-1 equal Rd, while the net cash flow equals $n(\psi) = \psi g(k+d) - Rd$, which it pays as dividends to the household. If the investment shock on date-1 is too adverse, i.e. if ψ is sufficiently low, the bank may not have enough resources to pay its depositors. In this case, the bank fails. We assume that the bank enjoys limited liability, so that shareholders cannot be asked for additional capital to rescue a failing bank. Instead, the assets of a failing bank are taken into custody by the government that covers any shortfall in liabilities using the deposit insurance scheme. We assume that bank failure imposes a cost Δ on the deposit insurance fund. Failure costs can arise due to bankruptcy resolution related losses, or due to forced sale of failed bank's assets by the deposit insurance agency. In our model, this cost gives rise to an externality because the mispriced deposit insurance allows the bank to over-borrow while not internalizing the expected social cost of failure it imposes.

Formally, the household chooses d in order to maximize its expected utility over the two periods:

$$U = \max_{d} \quad c + \beta \mathbb{E} c'(\psi)$$

$$s.t. \quad c = \bar{Y} - d \quad and \quad c'(\psi) = \bar{Y} + Rd + n(\psi) - T(\psi)$$

where $n(\psi)$ is the dividend paid by the bank to the household and $T(\psi)$ is the lump-sum tax. The bank chooses d to maximize the expected dividend it pays on date-1:

$$\max_{d} \quad \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f(\psi) d\psi$$

$$s.t. \quad k/d \geq \chi$$

The government's budget constraint is as follows:

$$T(\psi) = \begin{cases} 0 & if \quad \psi > \frac{Rd}{g(k+d)} \quad \text{ie the bank is solvent} \\ Rd - \psi g(k+d) + \Delta & if \quad \psi \leq \frac{Rd}{g(k+d)} \quad \text{ie the bank fails} \end{cases} \tag{1}$$

The first-order conditions (FOCs) are as follows:

$$\text{Household:} \qquad R = 1/\beta$$

[4]Alternatively, we could assume that the insurance scheme is funded via premium payments imposed on the bank. In either case, what matters is that the insurance scheme is mispriced, i.e. the premium is insensitive to the risk profile of the bank.

$$\text{Bank:} \qquad \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi = 0$$

This system of equations characterizes the competitive equilibrium (CE) in the baseline economy. While we cannot solve this system analytically, we will show that the CE is not constrained efficient (or second best). To this end, consider the problem of a constrained social planner that maximizes household welfare while choosing deposits on behalf of the bank, taking prices and the household's first order condition as given:

$$\max_{d} \quad c + \beta \mathbb{E} c'(\psi)$$

$$s.t. \quad R = 1/\beta \quad and \quad c + d = \bar{Y} \quad and \quad c'(\psi) = \bar{Y} + Rd + n(\psi) - T(\psi)$$

The last condition can be resolved using the expressions for $n(\psi)$ and $T(\psi)$:

$$c'(\psi) = \bar{Y} + \psi g(k+d) - \Delta \mathbb{1}\left(\psi \leq \frac{Rd}{g(k+d)}\right);$$

Next, the planner's objective can re-written explicitly as follows, where note that the first integral matches the bank's objective function:

$$\max_{d} \quad (1+\beta)\bar{Y} + \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f(\psi) d\psi +$$

$$\beta \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g(k+d) - Rd - \Delta\Big) f(\psi) d\psi.$$

Finally, the FOC of the planner's modified problem is as follows:

$$0 = \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi +$$

$$\underbrace{\beta \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi - \beta \Delta \frac{\partial \frac{Rd}{g(k+d)}}{\partial d} f\left(\frac{Rd}{g(k+d)}\right)}_{\text{Bank-failure externality}} \qquad (1)$$

The FOCs shows that there is a difference between the equation that characterizes d in the CE and that in the second-best. This difference can be attributed to a so-called bank-failure externality, which reflects the fact that the bank does not internalize the social cost of its failure, which is eventually borne by the taxpayer. The sign of the externality term can be determined using the method of contradiction. Assume that the externality term is positive. Then it must be that $\beta \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi > 0$ as the other part of the externality term is surely negative.[5] But then, $\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi > 0$. This is a contradiction since then the overall expression for the FOC becomes strictly positive. Thus the externality term must be negative. The implication of this is that

[5] Note that $\frac{Rd}{g(k+d)}$ is increasing in d.

$\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi > 0$. Now we know that d^{CE} (the solution in the CE) satisfies $\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi = 0$. But since $g(.)$ is concave, it must be that $d^{CE} > d^{**}$ where d^{**} solves the constrained planner's problem. This proves the following proposition:

Proposition 1. *The bank's capital ratio, defined as k/d, is smaller in the competitive equilibrium as compared to that in the constrained planner's problem.*

The above result also implies that $W^{CE} \le W^{**}$ where W^{CE} is the welfare in the CE and W^{**} is the second-best welfare. The natural question that follows is the implementability of the second best. To this end, we consider a benevolent regulator that sets a minimum capital-ratio requirement χ on the bank in order to maximize welfare. In implementing χ, the regulator faces the following trade-off. A higher χ forces the bank to shrink it's deposit based funding and reduce its failure probability, which has a welfare improving effect due to lower failure related cost. But at the same time, a higher χ depresses expected output, which has a welfare reducing effect. As such, a welfare trade-off arises. Let the optimal regulation be χ^o and the corresponding welfare be W^o. Then as long as the minimum capital-ratio requirement binds, the second best coincides with the solution to the benevolent regulator's problem. This is because the constrained planner chooses deposits on behalf of the bank, which is equivalent to a benevolent regulator choosing the bank's capital ratio when capital is a state variable.

Proposition 2. *The solution to the constrained planner's problem can be implemented via a minimum capital-ratio requirement.*

Finally, we note that the optimal capital requirement cannot achieve the first best, which is the solution to the problem of an unconstrained planner who does not need to go through a bank and can directly allocate resources. The intuition for this result is that in contrast to the constrained planner (or equivalently benevolent regulator), the unconstrained planner does not incur bank failure related costs, and thus does not face a welfare trade-off. Formally, the first best problem is:

$$\max_{d} \quad c + \beta \mathbb{E} c'(\psi)$$

$$s.t. \quad c + d = \bar{Y} \quad and \quad c'(\psi) = \bar{Y} + \psi g(k+d)$$

Assuming that the welfare in this case is denoted by W^*, we have $W^{CE} < W^o = W^{**} < W^*$.

3.1 Numerical simulation

The model does not admit an explicit analytical solution. We thus provide a numerical illustrate of the welfare trade-off capital regulation poses. The baseline parameter values and functional forms are presented in Table 1 (first block).[6] Numerical analysis confirms

[6]The parameter values are not based on a formal calibration of the model, which is beyond the scope of this paper. Nonetheless, we ensure that the qualitative insights are robust to perturbations in the parameter values. As regards the functional forms, while the exact choice does not matter, $g(s)$ must be decreasing-returns-to-scale, $\zeta(e)$ must be convex, and $p(e)$ must be monotonic and bounded.

Model	**Item**	**Description**	**Value**
Baseline (Two-period)	β	Discount factor	0.96
	g(s)	Expected pay-off from assets	s^{α}
	α	Pay-off exponent	0.9
	ψ	Random shock to asset pay-off	$F = \mathcal{N}(\mu, \sigma^2)$
	μ	Mean of ψ	1.4
	σ	Standard-deviation of ψ	0.2
	$\bar{Y}$	Household income	10
	k	Capital stock	1
	Δ	Bank-failure cost	$\in$ [0 2]
With uncertainty (Three-period)	$\zeta(e)$	Cost of exerting effort	$e^2/2$
	$p(e)$	Probability of being an H-type bank	$1 - 1/(1+e)$
	μ_H	Mean of ψ for H-type bank	1.4
	μ_L	Mean of ψ for L-type bank	1.35
With stress-tests	q_H	Pass probability of H-type bank	$\in$ [0.5 1]
	q_L	Pass probability of L-type bank	$\in$ [0.5 1]
With endogenous accuracy	$C(y)$	Welfare cost of accuracy	$(6 \times 10^-4)y$
	$q_H(y)$	Pass probability of H-type bank	$1 - 0.5/(1+y)$
	$q_L(y)$	Pass probability of L-type bank	$0.5/(1+y)$

Table 1: Parameter values and functional forms

that a higher requirement reduces bank failure probability, but also lowers expected output (see Figure 1). The net effect is an inverted U-shaped welfare profile. The optimal requirement is such that the marginal gains from lower bank failure related costs match the marginal loss from lower expected output. We confirm this intuition by showing that the optimal requirement increases as the cost of failure increases.[7]

4 Uncertainty about bank type

The baseline model provides a parsimonious framework where the bank-failure externality justifies capital regulation. We now extend the baseline model to study state-contingent capital requirements. We introduce an additional period to have a total of three dates: 0, 1, and 2. On the household side, date-0 is redundant, and consumption occurs on dates 1 and 2 only. On the bank side, we assume that there is uncertainty about it's type on date-1. The bank can be high (H) or low (L) type, depending on the expected return on it's assets on date-2, so that $\mu_H > \mu_L$. The probability p with which the bank is of H-type depends on the effort e it exerts on date-0. The cost of exerting this effort is $\zeta(e)$. Once the bank's type – or it's state – is revealed on date-1, it receives a fixed capital endowment, and raises deposits while satisfying the capital requirements. Regulation is bank-type specific, or state-contingent more generally, and is denoted by $\chi_s, s \in \{H, L\}$. It is announced on date-0 to enable the bank to know the future regulation and choose its date-0 effort accordingly. The problem of the bank, set recursively, is as follows:

$$V(\mu_s, \chi_s) = \max_{d} \quad \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f(\psi; \mu_s, \sigma^2) d\psi \quad s.t. \quad k/d \geq \chi_s$$

$$\max_{e} \quad -\zeta(e) + \beta\Big(p(e)V(\mu_H, \chi_H) + (1 - p(e))V(\mu_L, \chi_L)\Big)$$

The date-0 FOC reveals the determinant of the bank's effort, namely the *wedge* (say ω) between its value in the two states on date-1:

$$-\zeta'(e) + \beta p'(e) \underbrace{\Big(V(\mu_H, \chi_H) - V(\mu_L, \chi_L)\Big)}_{\omega} = 0$$

To see how the effort changes as this wedge increases, we take the total derivative of the FOC with respect to ω:

$$-\zeta''(e)\frac{de}{d\omega} + \beta p''(e)\omega\frac{de}{d\omega} + \beta p'(e) = 0$$

[7] The cost of bank failure has a macro as well as micro underpinning. During the global financial crisis, the U.S department of the Treasury authorized an amount of \$475 billion under the Troubled Asset Relief Program to stabilize the financial system. This amounts to 3.2% of the GDP in 2008 and can be thought of as the macroeconomic cost of bank distress. Bank distress or failures also occur during non-crisis periods. According to the Federal Deposit Insurance Commission (FDIC), there have been 25 bank failures during the last five years, and the estimated loss in each case has varied from less than 2% to more than 15% of the failed bank's assets. We thus set Δ to about 10% of the bank's assets in our model.

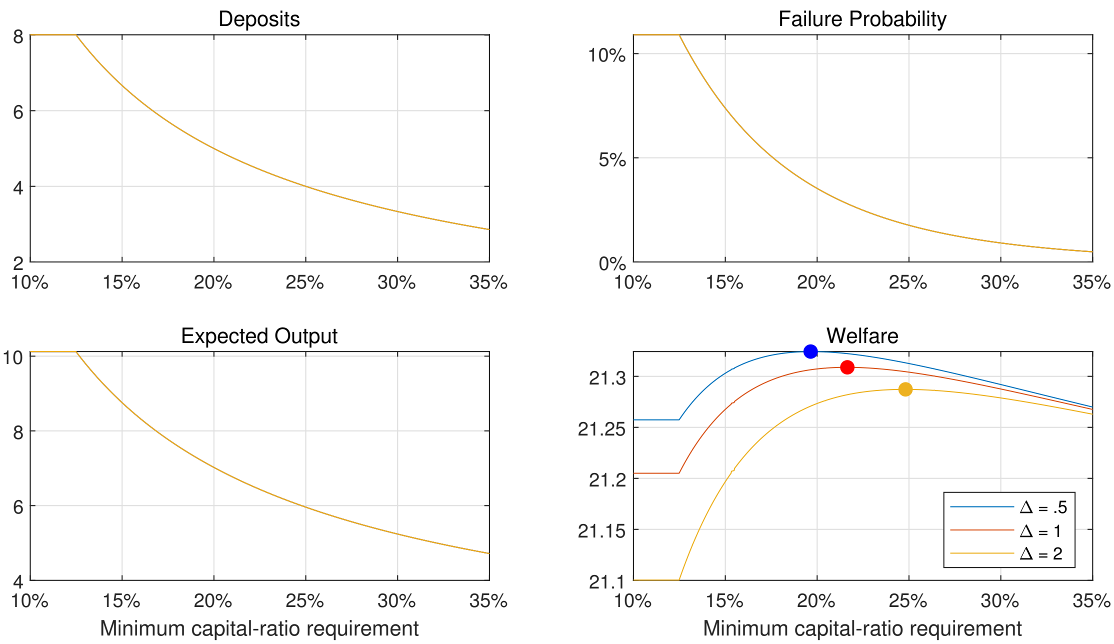

Figure 1: Bank response to an increase in minimum capital-ratio requirements, and the welfare maximizing regulation for varying levels of bank failure costs.

This implies that $\frac{de}{d\omega} > 0$ as long as $\zeta(.)$ is convex and $p(.)$ is increasing and concave, both of which are realistic assumptions. This proves the following proposition:

Proposition 3. *The bank exerts more effort when the relative value of being a high type compared to a low type is greater.*

Now we consider two possible information structures – depending on whether or not the regulator can observe the bank's type on date-1.

4.1 Full information case

In this case, the regulator can implement a state-contingent policy (χ_H, χ_L). Focusing on time-consistent policies, the optimal state-contingent policy on date-0 is (χ_H^o, χ_L^o) where χ_s^o is the optimal (ex-post) policy on date-1 in state s.[8] To solve this problem, we adopt parameter values and functional forms as shown in the second block of Table 1. We find that $\chi_L^o > \chi_H^o$. Intuitively, for any given d, an L-type bank not only poses a lower expected output, but is also more likely to fail. As a result, it poses a greater externality. This can be seen formally in equation (1), where as μ decreases, $f(q)$ increases for $q << \mu$. Since the integrand of the first part of the externality term is negative, this implies that the

[8]Note that (χ_H^o, χ_L^o) may not be the optimal policy on date-0. For instance, by announcing a policy (χ_H, χ_L) such that $\chi_H < \chi_H^o$ and $\chi_L > \chi_L^o$, a regulator could increase the wedge in the values of the bank in the two states and elicit greater effort from the bank. However, such a policy will not be credible because the regulator will have an incentive to deviate to χ_s^o in state s on date-1. A discussion of such time-inconsistency issues is beyond the scope of this paper.

externality term increases in magnitude as μ decreases. A larger externality rationalises a smaller d^{**}, or equivalently, a stricter regulation.

4.2 Information asymmetry case

Now we consider the case where the bank's type is its private information. We abstract away from the possibility that the bank can credibly communicate its type to the regulator (this would be equivalent to the full information case). As a result, the regulator cannot impose a state-contingent policy, and must adopt a state-independent capital requirement χ^o that would be applicable to the bank on date-1 irrespective of its type. The optimal χ^o is given as the solution to the following problem:

$$\max_{\chi} \quad \beta p(e) U_H + \beta(1 - p(e)) U_L$$

where U_s is the household's expected lifetime utility in state s on date-1. Numerical analysis reveals that $\chi_L^o > \chi^o > \chi_H^o$. Intuitively, when there is information asymmetry, the regulator chooses a middle-ground relative to the full information case.

4.3 Introducing stress tests

Stress tests allow regulators to gather information about those bank characteristics that are not necessarily reflected in their financial accounts or regulatory filings. A case in point is the Comprehensive Capital Analysis and Review (CCAR) exercise in the US which enables supervisors to identify the banks that are more susceptible to a weak macro-financial scenario – or banks that are relatively weak. Identifying such banks enables regulators to tailor the baseline regulatory requirements in a way that it is better aligned with banks' risk profiles. In a nutshell, stress tests enable bank-specific, or more generally state-contingent policy when there is information asymmetry.

To incorporate stress tests in our model, we adopt a parsimonious approach – one that captures the spirit of testing while not complicating the model unnecessarily. Specifically, we assume that once uncertainty is resolved on date-1, the stress test delivers a noisy signal η about the bank's type to the regulator. The test differentiates between H and L type banks by providing a signal distribution F_H of H type banks that dominates (in the first order stochastic dominance (FOSD) sense) the signal distribution F_L of L type banks. Depending on it's preferences for true- and false- positive and negative rates, the supervisor uses a signal cutoff η^c above (below) which the bank is considered pass (fail) and is deemed to be of the H (L) type. Then, the probability that an H type bank passes the test is given as $q_H = 1 - F_H(\eta^c)$, and the same for an L type bank is given as $q_L = 1 - F_L(\eta^c)$. Moreover, $F_H \succcurlyeq_{FOSD} F_L \implies q_H > q_L$.

Next we assume that a bank that passes the stress test faces no penalty and is allowed to operate at the pre-announced capital requirement χ^o, a failed bank's requirements are increased by $x \geq 0$. After this, the economy proceeds as before, i.e. the bank receives a capital endowment k, raises deposits d, invests in risky assets, and potentially fails on

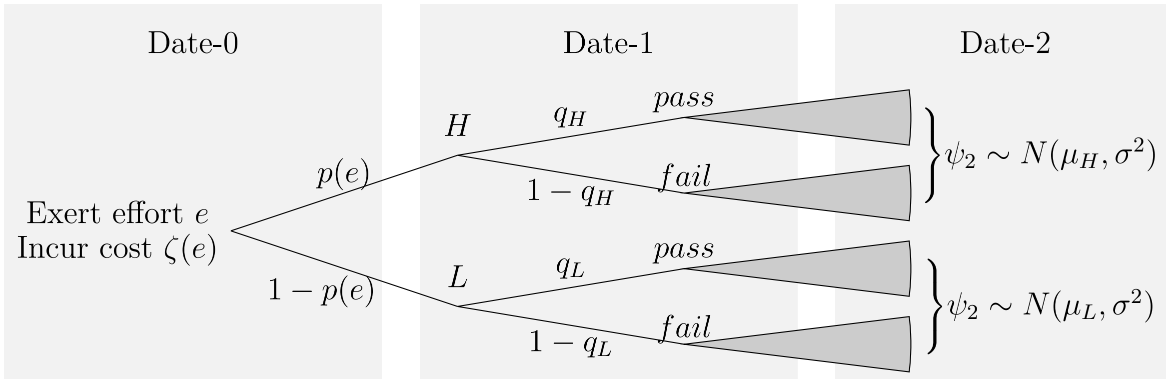

Figure 2: The timeline of events when there is information asymmetry about the bank's type, and stress tests serves as a policy tool to (partially) overcome the information friction.

date-2. See Figure 2 for the timeline of events. The date-0 problem of the bank is:

$$\max_e \quad -\zeta(e) + \beta p(e) \underbrace{\left(q_H V(\mu_H, \mathcal{X}^o) + (1-q_H) V(\mu_H, \mathcal{X}^o + x)\right)}_{\mathbb{E}V_H} +$$

$$\beta(1-p(e)) \underbrace{\left(q_L V(\mu_L, \mathcal{X}^o) + (1-q_L) V(\mu_L, \mathcal{X}^o + x)\right)}_{\mathbb{E}V_L}$$

As in the case without stress testing, the effort the bank exerts increases with the *expected* value function wedge $\mathbb{E}V_H - \mathbb{E}V_L$.

The question that arises then is about the optimal penalty x. The choice of x is subject to the following trade-off. On the one hand, it enables the regulator to re-optimize the regulatory requirement based on new information. This increases welfare ex-post. On the other hand, it affects the bank's behaviour ex-ante – depending on the accuracy of the stress test, the penalty can increase or decrease the effort the bank exerts. The optimal penalty solves the following problem:

$$\max_x \quad \beta p(e) \left(q_H U_{H,pass} + (1-q_H) U_{H,fail}\right) + \beta(1-p(e)) \left(q_L U_{L,pass} + (1-q_L) U_{L,fail}\right)$$

where $U_{s,r}$ is the utility of the household in state $s \in \{H, L\}$ and r denotes whether the bank passed of failed the test. To characterize the optimal x, we consider the following cases.

- *Perfect stress-test:* Test exactly identifies the type of the bank, so that $q_H = 1$ and $q_L = 0$. In this case, a higher x does not affect $\mathbb{E}V_H$, but decreases $\mathbb{E}V_L$. As a result, the bank increases effort as penalty increases. At the same time, the ex-post welfare increases as long as $x \leq \mathcal{X}^o_L - \mathcal{X}^o$. Beyond this threshold, the effective regulation, namely $\mathcal{X}^o + x$ is higher than $\mathcal{X}^o_L$, which is sub-optimal.

- *Imperfect stress-test:* Test results are noisy, so that an H type bank can fail the test ($q_H < 1$) and an L type bank can pass the test ($q_L > 0$). In this case, a higher x decreases both $\mathbb{E}V_H$ and $\mathbb{E}V_L$ because both H and L type banks can fail the test

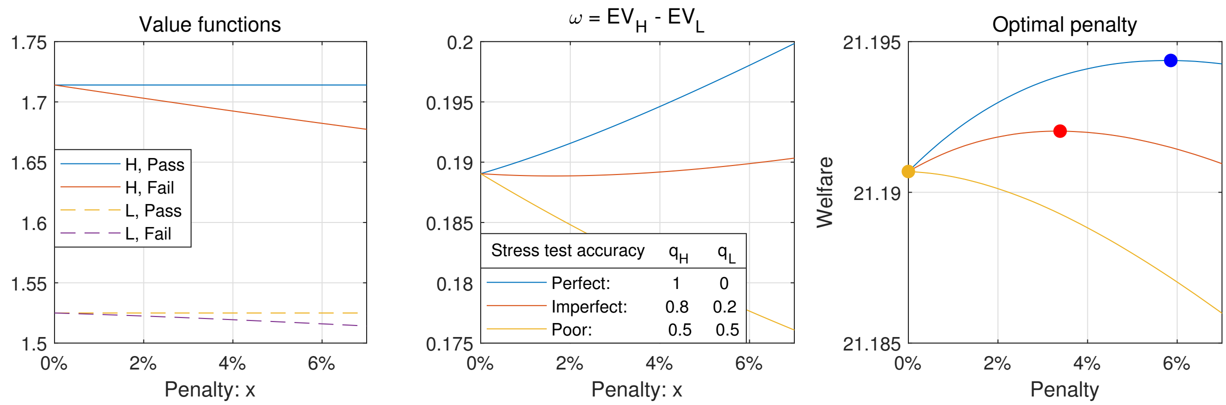

Figure 3: First panel shows the value of the bank in various cases for varying amounts of the penalty. Second panel shows how the expected value wedge changes in response to the penalty for different levels of accuracy of the stress test. Third panel documents the correspondingly optimal penalties.

and be penalized. Crucially, the difference between $\mathbb{E}V_H$ and $\mathbb{E}V_L$ can also decrease. This is because failing the stress test is more costly for an H bank whose assets are more profitable and that would prefer to assume more (rather than less) assets per unit of capital compared to an L bank. As a result, the bank on date-0 would exert lower effort as being an $H-$ relative to an L-type on date-1 is less attractive. This increases the probability that the bank is of the L-type on date-1, and thus expected output declines. On the net, while the penalty does increase welfare ex-post, it can create welfare reducing incentives ex-ante.

Figure 3 provides a numerical illustration of these insights based on parameter values in the first three blocks of Table 1. To summarize, the optimal penalty depends on the accuracy of stress tests – more the accuracy, larger the penalty that regulators could optimally impose. This result highlights a crucial trade-off for policymakers. On the one hand, penalizing the banks that fail the stress tests can induce them to improve their type (eg reduce their riskiness). However, on the other hand, the reverse is possible when there is uncertainty about the accuracy of the stress tests. As such, imposing limits on how state-contingent policy should be is desirable when the states are not observable accurately.[9]

4.4 Endogenous accuracy

Thus far, we take as given the accuracy of the stress test as captured by the probability with which an H (L) type bank passes (fails) the test. In practice, regulators can choose to design more or less accurate stress tests. Yet, they face trade-offs. On the one hand, as the preceding analysis shows, a more accurate test can improve the implementation

[9]By delivering this insight, our paper formalises a remark made my James Bullard, President of the Federal Reserve Bank of St. Louis in the context of quantitative easing. The essence of his remark was that while state-contingent policies are generally desirable, they work well when the states on which policy is contingent are known. See this article for a coverage of his comment.

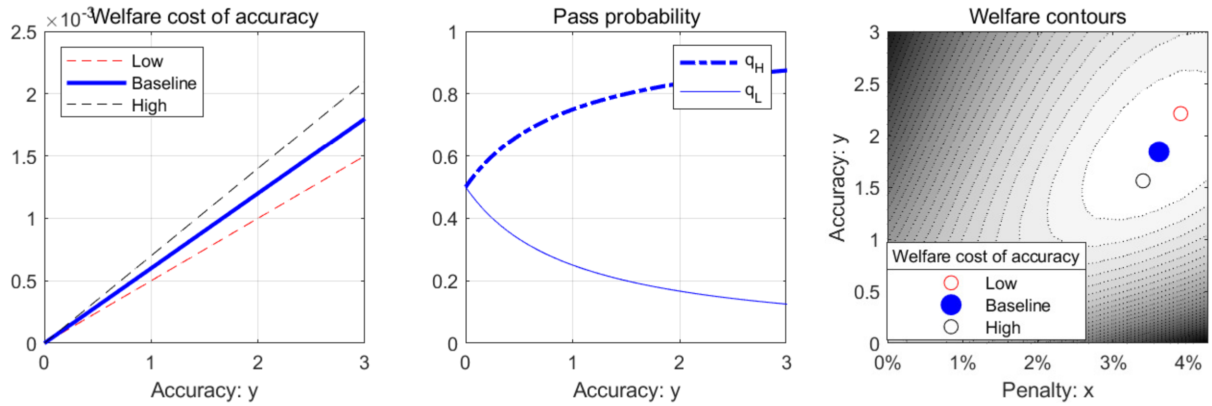

Figure 4: First panel plots alternative cost functions for accuracy $C(y)$. Second panel plots the various pass and fail probabilities as a function of accuracy. Third panel reports the jointly optimal accuracy and penalty for varying degrees of cost of accuracy. Note: Welfare contours are plotted for the baseline case only.

of state-contingent policy and thus improve welfare. On the other hand, designing and executing an increasingly accurate stress test can be costly for the regulator. Moreover, such a test would have to be more comprehensive and intrusive, and thus costly for banks as well – think of a very extensive asset quality review. To study this trade-off, we consider the problem of a regulator who chooses the optimal combination of penalty x and accuracy $y \geq 0$. For brevity, we assume that accuracy entails a welfare cost $C(y)$ and relates to the pass and fail probabilities as follows: $q_H(y) \uparrow 1$ as $y \to \infty$, and $q_L(y) \downarrow 0$ as $y \to \infty$.[10] Figure 4 illustrates the result of this extension based on the parametrization shown in the fourth block of Table 1. As the cost of accuracy increases, the regulator must optimally work with less accurate stress tests, and at the same time, revise downwards the penalty it imposes on failings banks.[11]

5 Conclusion

We study the trade-offs associated with state-contingent policies when the states are not well known. A case in point is the post-crisis reforms that impose specific requirements on banks based on their characteristics, some of which may not be fully observable to regulators. While stress tests do provide additional information about banks' health, the precision of such information is not guaranteed. As a result, banks may be misclassified, and face laxer or stricter regulation relative to the full information case. The possibility of misclassification can generate adverse incentives ex-ante. As such, while stress tests can improve welfare ex-post by reducing information frictions, poorly designed stress tests can generate adverse incentives and mitigate any potential welfare gains. We build a parsimonious model of bank capital regulation to formalize this trade-off. We show that

[10]The cost of a more accurate stress test can be micro-founded by treating it as an operating expense for the regulator and for the bank. We do not expect this change to alter the qualitative results.

[11]This result is robust as long as $C(y)$ and $q_H(y)$ are increasing, and $q_L(y)$ is decreasing.

when stress testing is sufficiently imperfect, the effort that banks exert in order to improve their risk profile, declines as the penalty for failing the test increases. Thus, while a penalty increases the ex-post welfare by aligning regulation to banks' types, it can decrease welfare overall due to the adverse ex-ante incentives it generates. As stress-tests become more precise, we show that the optimal penalty increases. Taking accuracy of the stress test to be endogenous, we show that the optimal accuracy and penalty combination increase hand-in-hand as the cost of accuracy decreases.

The model is amenable to further extensions. Consider, for instance, randomized state-contingent policy. A case in point is an argument often put forth by regulators that there is a need to maintain an element of surprise to avoid pre-positioning by banks. The welfare effects of surprise or extraneous noise in stress tests is not obvious because while it can limit the scope for gaming by banks, higher regulatory uncertainty can weaken the link between effort banks exert and their performance in the stress test. This can make banks exert less effort towards improving their risk-return profile. The current model can be used to study this trade-off by making regulation a *continuous* (as opposed to a discrete) function of the signal received as part of the stress test.

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