Limits of stress-test based bank regulation*

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Abstract

We develop a model to characterise the optimal bank-specific capital requirement based on supervisory risk assessments such as stress-tests. In the absence of such assessment tools, the regulator sets the same requirement across banks. Risk-assessment provides a signal about banks' types, and enables bank specific capital surcharges, which can improve welfare. Yet, noisy assessments can lead to excessive (or insufficient) regulation for some banks. Moreover, since higher requirements are disproportionately more costly for a bank with a better risk-return profile, noisy assessments can hamper banks' ex-ante incentives to improve their profile. This leads to riskier banks, and can decrease welfare. We show that when accuracy is below a threshold, no surcharge must be imposed. Otherwise, the optimal surcharge increases non-linearly with assessment accuracy.

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1 Introduction

The great financial crisis revealed that banks, especially large and complex ones, are opaque [Gorton, 2009]. As a result, regulators in many countries are now increasingly relying on risk assessment tools, such as supervisory stress-tests, to learn about bank-specific risk exposures. Such tools complement standard financial disclosures in informing regulators about banks' riskiness [Morgan et al., 2014] and help align baseline capital requirements with individual banks' risk profiles.[1] Empirical evidence, however, suggests that supervisory assessments may not always be able to correctly identify bank-specific risks [Acharya et al., 2014; Philippon et al., 2017; Plosser and Santos, 2018].

Despite empirical evidence of potentially inaccurate risk assessments, there is limited discussion in the literature on how capital requirements must be set when supervisory risk assessment – a key input to the capital regulation process – is noisy. Our goal in this paper is to help fill this gap. We develop a tractable model of how banks respond to capital requirements that are based on a potentially noisy signal about their riskiness. We use the model to derive the optimal capital requirement as a function of risk assessment accuracy. Finally, we study the trade-offs a regulator faces in making risk assessment more accurate.

The key players in our model are a banker and a regulator. The banker runs a bank that takes deposits from the household and invests in a risky project. The risk-return ratio on the project can be high or low, depending on the bank's *type*, which in turn depends on the effort it exerts ex-ante. A mis-priced deposit insurance combined with limited liability induce the bank to over borrow relative to the social optimal.[2] In turn, this rationalises a minimum capital-ratio requirement in our model, and allows us to study the welfare

[1] In the U.S., capital surcharges (among other requirements) are determined on the basis of stress-test results. In the Euro Area, stress-tests conducted by the European Banking Authority (EBA) are a crucial input into the Supervisory Review and Evaluation Process (SREP) which entails capital planning, reporting, and governance requirements tailored to individual banks.

[2] Typical reasons for a mis-priced deposit insurance include the inability of the insurer to observe banks' risk profiles or impose risk-sensitive premium payments. See Flannery et al. [2017] for elaboration.

implications of counterfactual policies.[3]

We begin by assuming that the regulator cannot observe the bank's type, which means that it may only impose a uniform requirement across banks. We then introduce a risk-assessment tool, such as a stress test, that provides a potentially inaccurate signal to the regulator about the bank.[4] Based on a signal cutoff, the bank is *deemed* as low or high type. This enables the regulator to impose a bank-type specific capital surcharge on top of the uniform requirement.

In imposing bank-specific requirements, the regulator faces the following trade-off. On the one hand, risk assessment helps overcome (some) information frictions and align regulation with individual banks' risk profiles. This improves welfare. On the other hand, a noisy risk assessment can lead to inefficiently low or high requirements for some banks, generate moral hazard issues, and lead to welfare losses. The intuition is as follows. Since higher capital requirements are more *costly* for a bank with a better risk-return profile, misdirected requirements can distort banks' ex-ante incentives by lowering the returns to exerting effort towards improving their risk-return profile.[5] Our model enables us to study the policy implications of this trade-off.

Our main contribution is to derive the optimal relationship between the accuracy of supervisory risk assessment and capital requirements in the presence of the moral hazard issue discussed above. We show that the welfare maximising mapping from assessment accuracy to capital requirements has three phases. When risk assessment is sufficiently

[3] A large literature provides several rationales for capital-ratio requirements, such as fire-sale externalities [Kara and Ozsoy, 2020], implicit government guarantees [Nguyen, 2015], moral hazard issues [Christiano and Ikeda, 2016; Gertler and Kiyotaki, 2010], and household preference for safe and liquid assets [Begenau, 2020]. The approach in this paper is related to that of Kareken and Wallace [1978], Santos [2001], and Van den Heuvel [2008] who show that over-borrowing, led by mis-priced deposit insurance or otherwise, justifies capital regulation.

[4] Stress-tests are one of the several ways in which regulators can obtain a signal about specific characteristics of banks. There are, indeed, other micro-prudential and supervisory tools that may provide similar signals and thus be subject to similar trade-offs that we model in the context of stress-tests.

[5] A large literature studies various adverse effects of capital requirements on banks' behavior. This includes studies such as Koehn and Santomero [1980]; Kim and Santomero [1988]; Rochet [1992]; Prescott [2004]; Gale et al. [2010]. We discuss how our paper relates to these studies in the literature review.

noisy, any bank-specific capital surcharge lowers welfare, and thus the optimal surcharge is zero. For intermediate levels of accuracy, we show that the optimal surcharge increases with accuracy, but is still smaller than what the full information benchmark would imply.[6] In case of a sufficiently accurate stress-test, the surcharge has a strong disciplining effect in terms of eliciting greater ex-ante effort from banks, and accordingly the optimal surcharge is closer to the full information case.[7]

Next, we consider the problem of a regulator who – in addition to choosing the optimal surcharge – can redesign the supervisory assessment to improve its accuracy. We assume that redesigning the assessment entails a (social) cost, say because of higher supervisory burden on both the regulator and the banks. We consider two cases depending on which dimensions of assessment accuracy – namely the false positive (high-type bank deemed low-type) and false negative (low-type bank deemed high-type) rates – the regulator can adjust.

In the first case, we assume that the regulator can reduce one or both of the false positive and false negative rates. In this case, as the cost of improving test accuracy becomes smaller, the regulator optimally increases the surcharge. In the second case, we assume that it is not possible to improve one *error* rate without worsening the other.[8] Herein, as the cost of reducing one error rate – say false positive – goes down, the regulator optimally chooses a lower false positive rate. However, unlike in the previous case, this shift may not support a higher surcharge because a lower false positive rate can only be achieved at the expense of a higher false negative rate. As a result, a dichotomy between

[6] Our paper supports the remarks made by Mark Zelmer (Deputy Superintendent, OSFI Canada) in 2013 in the context of risk-sensitivity of capital requirements. Relatedly, it formalises the intuition James Bullard (President of the Federal Reserve Bank of St. Louis) had in the context of quantitative easing – while state-contingent policies are generally desirable, they work well when the states on which the policy is contingent are known.

[7] Consistent with the empirical evidence in Delis and Staikouras [2011], our model shows that improving the quality of supervision by making stress-tests more accurate can enhance the disciplining role of capital regulation.

[8] This is typically the case when the regulator can only adjust the signal-cutoff that distinguishes between a high and low type bank.

test accuracy and the optimal surcharge arises.

In an extended version of the model, we study two additional policy trade-offs associated with risk assessments: disclosure of test results, and the role of failure costs associated with too-big-to-fail banks. We show that disclosure of test results can worsen regulatory trade-offs outlined in the baseline model. When the assessment is sufficiently accurate in identifying bank types, disclosures improve market discipline and facilitate the use of capital surcharges. Yet, when tests are less accurate, disclosures can induce greater risk-taking by banks, and thus place further limits on the use of surcharges. The regulatory trade-offs are also aggravated when bank failures are more costly, such as in the case of too-big-to-fail banks. We show that in this case, not only is the optimal baseline capital requirement stricter, the optimal surcharge for a given level of accuracy is also higher.

To illustrate our analytical results, we calibrate the parameters of the model using data on U.S. banks. Numerical computations allow us to fully characterise the relationship between accuracy and optimal surcharge. Consistent with the theoretical predictions, numerical simulations show that the surcharge is zero if the accuracy is below a threshold, and increases non-linearly with accuracy otherwise.

We conclude our discussion by alluding to potentially noisy assessments in the 2020 Dodd-Frank Act Stress Test in the U.S. that was conducted right before the Covid-19 crisis. Our observations complement existing studies on inaccuracies in risk assessment of banks. We document that the cross-sectional variance in test-driven changes in banks' CET1 ratios is much higher than the observed changes during the Covid-19 crisis, and that the two do not correlate. This observation raises the question of whether the stress-test based capital surcharges were too high or low for some banks.

Related literature

Our research question is motivated by studies highlighting limitations of supervisory risk assessments, stress-tests in particular. Acharya et al. [2014] find that in the 2011 European stress-test, the assessment of banks' risk was not in line with their realized risk following the disclosure of test results. For the 2014 stress-test conducted by the European Banking Authority, Philippon et al. [2017] find that while model-based losses are good predictors of realized losses, banks headquartered in countries with weak banking system have higher realized losses compared to losses predicted by the test. Similarly, Frame et al. [2015] show that stress-tests conducted by the U.S. Office of Federal Housing Enterprise Oversight in the pre-GFC period failed to detect risks on the balance sheets of Fannie Mae and Freddie Mac. More generally, Berger et al. [2000] show that supervisory assessments are generally less accurate than market indicators in predicting banks' future performances.[9]

Empirical evidence of potential inaccuracies in supervisory risk assessment has motivated a growing literature on aspects of assessment such as efficient information acquisition [Parlatore and Philippon, 2020], transparency [Leitner and Williams, 2020; Quigley and Walther, 2020], and disclosure [Goldstein and Sapra, 2014; Bouvard et al., 2015; Williams, 2017; Goldstein and Leitner, 2018; Orlov et al., 2018]. However, one aspect of supervisory assessments that has received less attention in the literature is the implication of potentially noisy assessments for how the *attendant* capital requirements must be set. This is despite a recognition of this issue in policy discussions [Zelmer, 2013; Powell, 2019]. In this paper, we hope to make progress in filling this gap.

A paper that closely shares our pursuit is Ahnert et al. [2020] which shows that the sensitivity of regulation to banks' types must depend on the precision of the signal generated by the risk assessment tool. Specifically, the authors show that beyond a level of accuracy, risk sensitivity of capital regulation should decrease with signal precision. Relat-

[9] Supervisory risk assessment is an inherently hard pursuit, and some inaccuracies are inevitable. These can stem from noisy bank-level inputs used in assessment models [Ong et al., 2010], limits of internal risk models of banks [Leitner and Yilmaz, 2019; Plosser and Santos, 2018; Behn et al., 2016; Wu and Zhao, 2016], or limits of econometric models used by the regulators to predict bank losses [Covas et al., 2014]. It could also be that stress-tests do not fully take into account the endogenous reaction of banks to the stress event, and thus fail to provide an adequate risk-assessment [Braouezec and Wagalath, 2018]. Technical and computational glitches can also lead to noisy assessments. For example, in September 2020, the U.S. Federal Reserve Bank published corrections to its previously issued stress-test results [Fed, 2020].

edly, Morrison and White [2005] show that when the regulator's screening ability (or audit reputation) is lower, capital regulation, as a substitute to screening, must be tighter to compensate. By contrast, we show that when risk assessment, an input to capital regulation, is more noisy, higher capital requirements can lead to adverse incentives. In turn, this rationalises a less tight regulation in our model. The difference in our conclusions stems from the relative role of screening *vis-a-vis* regulation, and the fact that we allow banks to affect the probability that they face a capital surcharge, due to which regulation directly affects banks' ex-ante incentives.

More broadly, studies on state-contingent regulation are related to our analysis of bank-type dependent regulation – this is because bank-type can be interpreted as a *state*. For instance, Marshall and Prescott [2001] show that state-contingent fines on banks can increase welfare, but assume that the states are observable, unlike in our analysis. Lohmann [1992] shows that when future states are not fully known, it is sub-optimal to commit to a state-contingent policy. By comparison, while we share this insight, we allow the policy maker to choose the *degree* of state-contingency, and we characterise its optimal value.

A key element of our analysis is the modeling of how banks respond to capital requirements. To be sure, several papers have analysed this question before. An early work is by Koehn and Santomero [1980] who show that tighter capital requirements can lead some banks (modeled as portfolio managers) to become even more risky. In follow-up research, Kim and Santomero [1988] as well as Rochet [1992] show that this result disappears when risk-weights used to compute capital requirements are consistent with asset quality. By comparison, our analysis is based on the idea that risk assessment is inherently noisy, as a result of which capital requirements can lead to adverse incentives. While our headline conclusion resonates with that of Prescott [2004] where poorly executed supervisory audits can create adverse incentives for banks because they can disclose information strategically, or Gale et al. [2010] where higher capital can force banks to take more risk to achieve the required rate of return, the underlying mechanism in our paper is distinct. We show that a moral hazard issue arises as the cost of tighter regulation is greater for a high-type bank – indeed, in the presence of misdirected requirements, this can diminish a bank's incentives to improve its risk-return profile.

2 Model

Our goal is to analyse the welfare implications of capital requirements that are based on potentially noisy risk assessment of banks. To this end, we develop a model with the following main elements. First is a general equilibrium setup that enables us to capture the welfare effect of regulation. Second is a dynamic setup that allows us to study the effect of assessment based regulation on banks' ex-ante behavior. Third is a rationale for capital regulation – specifically, a social inefficiency that warrants regulatory intervention. Fourth is information frictions – i.e., the unobservability of a bank's type by the regulator – that justify the use of risk assessment. Accordingly, we consider an economy that lasts three periods (0, 1, and 2), and consists of a representative household, a banker whose decisions are socially inefficient and whose type is stochastic, a regulator that cannot (fully) observe the bank's type, and a government that runs a deposit insurance program.

Household The household is representative, and receives an unconditional income endowment $\bar{Y}$ on dates 1 and 2. On date-1, it decides how much to consume, c_1, and how much to deposit, d, in the bank.[10] Deposits are risk-free, and pay a gross return of R on date-2.

Banker The banker has a capital endowment of k on date-1. It runs a bank that issues deposits d to invest $k + d$ in a risky project that pays $\psi g(k + d)$ on date-2. $g(.)$ is a decreasing returns to scale (DRS) return function. ψ is an investment shock whose density

[10] A time subscript is used only for those quantities that are relevant on multiple dates. For instance, since d is only chosen once, on date-1, a time subscript is omitted.

f_s depends on the banker's type s on date-1, which can be high (H) or low (L). Specifically, we assume that while both types face the same standard deviation of ψ, namely σ, the high-type bank has a higher expected return, $\mu_H > \mu_L$, so that a high-type bank has a higher *risk-adjusted return*. The probability p with which the bank is of high-type depends on the effort e the banker exerts on date-0. We assume $p(e)$ is increasing and concave. The cost of exerting effort is given as $\zeta(e)$, which we assume to be increasing and convex. The bank learns its type on date-1.

The bank's deposit liabilities on date-2 equal Rd, and thus the net cash-flow n equals $\psi g(k+d) - Rd$. When ψ is sufficiently high and the bank is solvent, the entire cash-flow is paid as dividends to the banker. However, when ψ is low enough so that the cash-flow is negative, the bank fails and banker receives null. We assume that the banker only consumes on date-2, and that it has limited liability, so that it cannot be asked for additional capital to rescue a failing bank. Instead, the government takes the bank into receivership.

Government The government runs the deposit insurance scheme and ensures that depositors are fully protected against bank failure. When a bank fails, the government liquidates its assets, and covers any shortfall in the failed bank's liabilities. To fund the scheme, the government imposes a lumpsum tax T on the household. We assume that the insurance scheme is mis-priced – ie insensitive to the risks banks take – which, as we prove later, leads to a social inefficiency.[11] The government runs a balanced budget.

Regulator The regulator is benevolent, i.e. it strives to maximise the joint welfare of the household and the banker. On date-0, it announces the minimum capital-ratio requirement χ that the bank must satisfy on date-1. However, we assume that the regulator cannot *observe* the bank's type on date-1.[12] In the baseline economy, as such, it must announce

[11] The reason for introducing an inefficiency in our model is to rationalise capital requirements. A mis-priced deposit insurance is not the only way to do so, but it is a relatively simple method that helps keep our model tractable. Another paper to have taken this route is Van den Heuvel [2008].

[12] In reality, regulators do have some knowledge about banks' characteristics (such as via regulatory filings). We assume that the observable characteristics are embedded in the return function $g(.)$ of the

a requirement that does not depend on banks' type, i.e. applies universally to both types of banks on date-1. In the economy with stress-tests, the regulator is able to obtain a noisy signal about the bank, and classify it as a high- or low-type depending on whether it passes or fails the test. The regulator then imposes a surcharge x for failing the stress-test, effectively imposing a bank-type specific requirement $\chi_s, s \in \{H, L\}$.

Recursive formulation We now formally setup the problem statements of the agents in the economy. The household chooses d on date-1 to maximize its expected utility over dates 1 and 2, where β is the discount factor:

$$U = \max_{d} \quad c_1 + \beta \mathbb{E} c_2 \quad s.t. \quad c_1 = \bar{Y} - d \quad and \quad c_2 = \bar{Y} + Rd - T. \tag{1}$$

The banker chooses e on date-0 which determines the probability of being an H-type on date-1:

$$[Date - 0]: \quad \max_{e} \quad -\zeta(e) + \beta \Big(p(e) V_H(\chi) + (1 - p(e)) V_L(\chi) \Big). \tag{2}$$

where $V_s(\chi)$ is defined in equation (3). The bank of type $s \in \{H, L\}$ chooses d on date-1 to maximize the expected dividend it pays on date-2:[13]

$$[Date - 1]: \quad V_s(\chi) = \max_{d} \quad \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\underbrace{\psi g(k+d) - Rd}_{n} \Big) f_s(\psi) d\psi \quad s.t. \quad \frac{k}{\chi} \geq d. \tag{3}$$

The lower limit on the integral is the ψ cut-off – call it ψ_c – below which the bank fails (and no dividends are paid). χ is the minimum capital-ratio requirement.[14] The government's

bank while *type* simply summarizes the unobservable characteristics. Furthermore, we assume that the bank cannot credibly communicate its type to the regulator, except via its performance in a stress-test.

[13] We implicitly already assume that the deposit market clearing condition holds, and thus use the same d for the household's and the bank's choice of deposits.

[14] The requirement can vary across banks depending on their performance in the stress-test. This case is discussed later.

budget constraint is as follows:

$$T = \begin{cases} Rd - \psi g(k+d) & \text{If the bank fails i.e. } \psi \leq \frac{Rd}{g(k+d)} \\ 0 & \text{Otherwise} \end{cases} \tag{4}$$

3 Qualitative Analysis

We begin by assessing the equilibrium conditions in the baseline economy. We then characterise – as a benchmark – the optimal regulation in the absence of stress tests. Finally, we analyse the optimal capital surcharge based on stress-test results, including when results are disclosed, and when bank failure is socially costly.

3.1 The baseline equilibrium

The first-order condition (FOC) of the bank's problem on date-0 shows that the effort the bank exerts depends on the *wedge*, say ω, between the value of being a high- as opposed to low-type on date-1:

$$-\zeta'(e) + \beta p'(e) \underbrace{\Big(V_H(\chi) - V_L(\chi)\Big)}_{\omega} = 0 \tag{5}$$

To see how the effort changes as the wedge increases, we take the total derivative of Equation 5 with respect to ω, from where it is straightforward to note Lemma 1:

$$-\zeta''(e)\frac{de}{d\omega} + \beta p''(e)\omega\frac{de}{d\omega} + \beta p'(e) = 0 \tag{6}$$

Lemma 1. *If $\zeta(.)$ is increasing and convex, and $p(.)$ is increasing and concave, then the bank exerts more effort when the difference in the value of being a high type compared to a low type increases, i.e. $de/d\omega > 0$.*[15]

[15] The result does not depend on the convexity of the cost function; a linear $\zeta(.)$ would suffice. Yet, an

It's intuitive to see from equation 5 why effort would increase with the wedge ω. As the relative value of being a high-type bank increases, the marginal benefit of effort increases while the marginal cost is unaffected. Lemma 1 underscores that the minimum requirement (χ) affects the wedge ω by impacting the value of the bank on date-1. As such, the minimum requirement is a key factor in bank's effort choice on date-0, and will shape the regulator's choice of optimal ex ante capital requirement as we show later in Section 3.3.[16]

As regards the date-1 FOCs, we have the following:

$$\text{Bank:} \qquad \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi - \Lambda_s = 0 \tag{7}$$

$$\text{Household:} \qquad R = 1/\beta \tag{8}$$

Note in the bank's FOC that Λ_s is the Lagrange multiplier on the regulatory constraint, and that two of the three terms which arise from a routine application of the Leibniz rule are equal to zero. The system of FOCs (5), (7), (8) and the government's budget constraint (4) together characterise the competitive equilibrium of the model economy for a given set of minimum capital-ratio requirements (χ_H, χ_L).

3.2 Optimal ex-post regulation

We now assess the efficiency of the equilibrium, and discuss the role that regulation could play in improving welfare. In this section, we focus on the date-1 economy, and turn to the date-0 economy (and the discussion of stress tests) in the next subsection.

Inefficiency of the equilibrium We compare allocations in an unregulated date-1 economy with a benevolent social planner's allocations. As such, for now we ignore the banker's date-0 problem ie it's effort choice (and return to this consideration later). Without loss

increasing marginal cost of effort is a realistic assumption to have.

[16] Lemma 1 is related to a similar result proven in Christiano and Ikeda [2016], but the channel through which regulation has an impact on the banker's effort is different.

of generality, refer to a banker of type s, where s could be high or low.

We consider a constrained social planner who maximizes the date-1 and date-2 equally weighted welfare of the household and the banker by choosing the level of deposit funding on behalf of the banker, taking as given the household's first order condition:

$$\max_{d} \quad c_1 + \beta \mathbb{E}(c_2 + n) \quad s.t. \quad R = 1/\beta; \quad c_1 = \bar{Y} - d; \quad c_2 = \bar{Y} + Rd - T$$

Recall that the banker does not consume on date-1, and note that $c_2 + n$ denotes the combined consumption of the household and the banker on date-2. Since the planner internalises the effect of choosing d on n and T, we can solve for $c_2 + n$ using expressions for n and T from equations (3) and (4) respectively:

$$c_2 + n = \bar{Y} + \psi g(k + d) \tag{9}$$

Next, we rewrite the planner's objective after plugging in the expressions for c_1, c_2, n, rearranging terms using the household's FOC, and segregating the expectation (i.e. the integral on $c_2 + n$) at the ψ cutoff for failure of the bank:

$$\max_{d} \quad (1+\beta)\bar{Y} + \underbrace{\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f_s(\psi) d\psi}_{\text{Banker's date-1 objective}} + \beta \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g(k+d) - Rd\Big) f_s(\psi) d\psi. \tag{10}$$

By segregating the integral into two parts, the first part matches the bank's objective function, and thus facilitates a comparison of bank's and planner's FOCs, as shown below:

$$\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi + \underbrace{\beta \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi}_{\text{Bank-failure inefficiency}} = 0. \tag{11}$$

Equation (11) uncovers a wedge between the planner's FOC and the bank's FOC in the unregulated economy (i.e. equation (7) with $\Lambda_s = 0$). This wedge stems from limited liability and a mis-priced deposit insurance. Because of limited liability, the bank does not internalise the losses corresponding to the left tail of the distribution of ψ – the part that corresponds to bank failure. And because of deposit insurance, the depositors do not charge a premium for risk of non-repayment of deposit proceeds post bank failure. The bank thus over-borrows. The planner, on the contrary, chooses the level of deposits taking into account the entire distribution of ψ. We refer to this wedge as the bank-failure inefficiency, which the following lemma characterises.

Lemma 2. *The bank's capital ratio, defined as k/d, is smaller in the competitive equilibrium as compared to that in the constrained planner's problem, i.e. the second best.*[17]

Proof. See Appendix A. ■

Implementability of the constrained efficient allocation That the competitive equilibrium exhibits an inefficiency implies that $W^{CE} \leq W^*$ where W^{CE} is the welfare in the competitive equilibrium and W^* is the second-best welfare. The question that follows is whether a regulatory intervention can help implement or approach the second best.

To this end, we consider a benevolent regulator who sets a minimum capital-ratio requirement $k/d \geq \chi_s$ on the bank in order to maximize welfare. In choosing χ_s, the regulator faces the following trade-off. A higher χ_s forces the bank to reduce deposit-based funding and accordingly its failure probability, which has a welfare improving effect due to a smaller bank-failure inefficiency. Yet, a higher χ_s depresses expected output, which has a welfare reducing effect.

In effect, the regulator's decision problem is very similar to that of a constrained planner. This is because choosing deposits on behalf of the bank to maximise welfare is equiva-

[17] The finding that the bank takes more leverage than what is socially optimal is not unique to this paper, nor is it our main contribution. Several other studies have related findings, such as Van den Heuvel [2008] and Christiano and Ikeda [2016], for instance. Our approach is to develop a relatively parsimonious model that has the mechanisms needed to study the welfare effects of stress-test based capital requirements.

lent to imposing a minimum capital-ratio requirement with the same objective when capital is fixed and the requirement is binding. This is formally seen by comparing equations (7) and (11). Indeed, the first terms are identical. And to the extent the Lagrange multiplier Λ_s on (i.e. the shadow cost of) the regulatory constraint in (7) is equal to the absolute value of the bank-failure inefficiency term in (11), the solution to the two equations must be identical. We note this result in the lemma below, and denote the optimal regulation for an s-type bank by χ_s^o.

Lemma 3. *The solution to the constrained planner's problem can be implemented via a minimum capital-ratio requirement.*[18]

Before turning to the date-0 problem, we document a result that will be useful later. It compares the optimal date-1 regulation for high- and low-type banks. Assume that the regulator can perfectly observe bank type.

Lemma 4. *The regulator optimally sets a higher ex-post requirement on the low-type bank as compared to a high-type bank, i.e.* $\chi_L^o > \chi_H^o$.

Proof. Consider the non dis-aggregated version of the planner's date-1 FOC – i.e. equation (11) – for both high- and low-type banks. This characterises the optimal level of deposits in each case.

$$0 \;=\; \int_0^\infty \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi \;=\; \mu_s g'(k+d) - R \quad s \in \{H, L\} \tag{12}$$

The total derivative of d with respect μ_s implies:

$$g'(k+d) + \mu_s g''(k+d)\frac{\partial d}{\partial \mu_s} = 0 \implies \frac{\partial d}{\partial \mu_s} > 0 \quad s \in \{H, L\} \tag{13}$$

[18] A capital-ratio requirement is not the only regulatory tool that can implement the second best. A tax (or a deposit insurance premium) that is a function of the balance sheet choice of the bank may also achieve the same objective.

This immediately implies that the optimal d is higher, or equivalently, the optimal χ^o is lower for a high-type bank. ■

Intuitively, for a given level of deposits, a low-type bank not only generates lower expected output, but is also more likely to fail. This underpins the stricter regulation for the low-type bank.

3.3 Optimal ex-ante regulation

The bank forms expectations and chooses its date-0 decisions based on date-1 requirements announced by the regulator on date-0.[19] However, because the bank's type on date-1 is its private information, the regulator cannot announce a type-specific requirement (such as χ_L^o and χ_H^o for low- and high-type banks respectively). As a result, the regulator must adopt a uniform capital requirement – say χ – which is applicable on date-1 irrespective of the bank's type. To characterize the optimal χ, we begin with the following result.

Lemma 5. *Assume that regulation χ binds for both bank types on date-1.*[20] *Then the effort the bank chooses to exert on date-0 decreases as χ rises.*

Proof. As shown in Lemma 1, the bank's date-0 effort e depends on $\omega = V_H(\chi) - V_L(\chi)$, i.e. the wedge between the value of being a high- versus low-type on date-1. The key then to proving this lemma is to characterise how regulation impacts ω.

$$\omega = \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f_H(\psi) d\psi - \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) f_L(\psi) d\psi$$

[19] We abstract away from time-inconsistency issues, and assume that regulatory announcements are credible.

[20] A concave yet sufficiently close to linear asset return function $g(.)$ would ensure that regulation always binds. In practice, banks hold a so-called management buffer beyond the minimum requirement, but this can be included in our framework as a constant on top of the minimum requirement.

where $d = k/\chi$. The derivative of ω with respect to χ gives:

$$\frac{\partial \omega}{\partial \chi} = -\frac{k\beta}{\chi^2}\left(\underbrace{\int_{\frac{Rd}{g(k+d)}}^{\infty}\Big(\psi g'(k+d) - R\Big) f_H(\psi)d\psi}_{\Lambda_H} - \underbrace{\int_{\frac{Rd}{g(k+d)}}^{\infty}\Big(\psi g'(k+d) - R\Big) f_L(\psi)d\psi}_{\Lambda_L}\right) \quad (14)$$

where Λ_s is the Lagrange multiplier on the regulatory constraint in the bank's problem.

To sign this expression, we proceed as follows. First note that since the (binding) regulatory requirement is the same for both types of banks, their deposit choices and thus the failure cutoffs ψ_c are also the same. Then let $\hat{F}_H$ and $\hat{F}_L$ be the distribution functions of ψ for high- and low-type banks, truncated below at ψ_c. Since $\mu_H > \mu_L$ (while the variances are the same), $\hat{F}_H$ *FOSD* $\hat{F}_L$, that is $\hat{F}_H(\psi) \leq \hat{F}_L(\psi)\ \forall\ \psi$. Finally, since $(\psi g'(k+d) - R)$ is an increasing function of ψ, it follows that:

$$\int \Big(\psi g'(k+d) - R\Big) d\hat{F}_H(\psi) - \int \Big(\psi g'(k+d) - R\Big) d\hat{F}_L(\psi) = \Lambda_H - \Lambda_L > 0.$$ [21]

In turn, this implies that $\frac{\partial \omega}{\partial \chi} < 0$. Then from Lemma 1 we know that $\frac{\partial e}{\partial \omega} > 0$, which completes the proof since:

$$\frac{\partial e}{\partial \chi} = \frac{\partial e}{\partial \omega}\frac{\partial \omega}{\partial \chi} < 0.$$

■

Lemma 5 captures a key insight of this paper. Because a high-type bank's assets are more profitable, the opportunity cost of stricter capital requirements is greater for this bank. As such, an increase from a given level of requirement leads to a greater decline in

[21] To prove this formally, consider continuous distribution functions G and H such that $\forall x, H(x) \leq G(x)$, and define $y(x) = H^{-1}(G(x))$. Then for any increasing function $w(x)$, $\int w(y(x))dH(y(x)) = \int w(y(x))dG(x)$. Next, note that $y(x) = H^{-1}(G(x)) \implies y(x) \geq x$ since $\forall x, H(x) \leq G(x)$. In turn, since $w(.)$ is an increasing function, $w(y(x)) \geq w(x)$. Thus, $\int w(y(x))dG(x) \geq \int w(x)dG(x)$. Indeed, intuitively, the *shadow cost* of the minimum capital-ratio constraint should be greater for a bank whose assets are *ceteris paribus* more profitable.

the expected value of the high-type bank than the low type bank. This, in turn, lowers the returns to exerting more effort. In contrast to the conventional wisdom that more skin-in-the-game via higher capital requirement can induce banks to become safer, our finding is that under information frictions banks might respond to stricter regulation by becoming more risky.

This insight thus points to an important trade-off the regulator faces while setting χ. Compared to no regulation ($\chi = 0$), a higher χ can improve welfare *ex-post* by mitigating some of the inefficiency associated with the bank's choices, especially in case of a low-type bank. Yet, a higher χ can reduce welfare due to its adverse impact on effort exerted *ex-ante*.

Before characterising the optimal ex-ante regulation, we note that the assumption that regulation binds for both bank types is not critical for Lemma 5. The case where regulation binds for only one bank – which has to be the high type bank since it chooses a lower capital ratio in the unregulated economy – leads to the same result because in that case $\Lambda_L = 0$. The case where regulation does not bind for any bank is not relevant nor interesting because we already showed that an inefficiency rationalises *some* regulation.

Proposition 1. *The optimal ex-ante requirement χ^o in the case where the regulator cannot observe the bank's type, lies between by the optimal ex-post requirement for low- and high-type banks, ie $\chi^o_L \geq \chi^o \geq \chi^o_H$.*

Proof. The problem of a benevolent regulator on date-0 when it cannot impose bank-specific requirements, is as follows:

$$\max_{\chi} \quad \beta p(e) U_H(\chi) + \beta(1 - p(e)) U_L(\chi) - \zeta(e)$$

Here U_s is the household's and banker's combined expected lifetime consumption utilities when the banker turns out to be of type s, while $\zeta(e)$ accounts for the banker's effort on date-0. We will prove the proposition via the method of contradiction. Let χ^o solve the above problem. Then, if $\chi^o > \chi^o_L > \chi^o_H$, it means that the requirement is more strict than the optimal requirement for both bank types, and thus a lower χ^o would improve welfare in case of each bank type, as well as the total expected welfare. Similarly, if $\chi^o_L > \chi^o_H > \chi^o$, it means that the requirement is more liberal than the optimal requirement for both bank types, and thus a higher χ^o would improve total welfare. ■

Intuitively, this proposition shows that when there is information asymmetry, the regulator chooses a *middle-ground* relative to the optimal bank-type specific requirements.

3.4 Mitigating information frictions via stress-tests

Supervisory assessments help gather information about banks' types. Mitigating some information frictions allows capital requirements to be better aligned to the banks' types. This is desirable as it can improve welfare.

We model supervisory assessment as a stress-test that delivers a noisy signal to the regulator about the bank's type. Based on the test outcome, the regulator deems the bank to be of the low or high type (see Figure 1 for the timeline). We assume that the probability that a high-type (low-type) bank is deemed high is q_H (q_L).[22]

The accuracy of the stress-test is fully captured by the tuple (q_H, q_L). Any test can thus be represented by a point in the set $[0, 1] \times [0, 1]$, as shown in Figure 2. In this format, $(1 - q_H)$ denotes the 'false positive' or Type-I error rate (high-type bank fails the test), while q_L is the 'false negative' or Type-II error rate (low-type bank passes the test). A

[22]The pass probabilities are rationalised as follows. We assume that the signal distribution, say Q_H, of high-type banks dominates (in the first order stochastic (FOSD) sense) the signal distribution Q_L of low-type banks. Depending on its preferences for true- and false- positive and negative rates, the supervisor uses a signal cutoff η^c above (below) which the bank is considered pass (fail) and is deemed to be of the high- (low-) type. Thus the probability that a high-type bank passes the test is given as $q_H = 1 - Q_H(\eta^c)$, and the same for a low-type bank is given as $q_L = 1 - Q_L(\eta^c)$. Moreover, $Q_H \succcurlyeq_{FOSD} Q_L \implies q_H > q_L$. Note that we do not model the regulator's preferences for Type-I and Type-II error rates that determine the signal cutoff η^c and thus the pass probabilities. This mapping is standard in the literature, and is typically based on the receiver operating characteristics (ROC) curve. In section 3.5 we assess the implications of a change in the signal cutoff for the optimal policy, and also the case where the regulator can incur a cost and improve the overall accuracy of the stress-test – i.e. increase the area under the ROC curve.

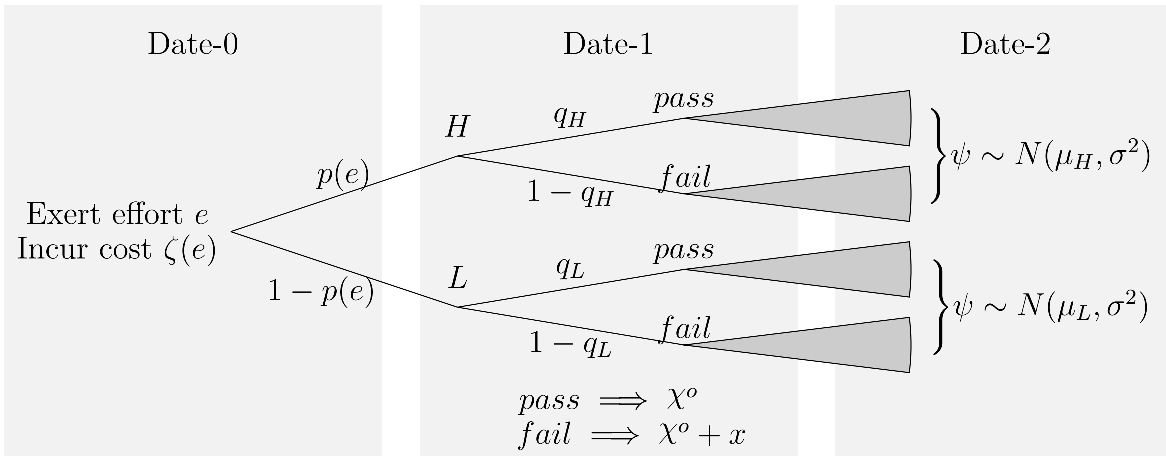

Figure 1: The timeline of events when there is information asymmetry about the bank's type, and stress tests serves as a tool to (partially) mitigate information frictions.

convenient benchmark, which is equivalent to the full-information case, is when $q_H = 1$ and $q_L = 0$, i.e. a *perfect* stress-test that exactly identifies the type of the bank. In all other cases, we refer to the test as *imperfect* because an H-type bank can fail the test ($q_H < 1$) or an L-type bank can pass the test ($q_L > 0$).

The regulator uses the outcome of the stress-test to adjust the baseline capital requirement χ^o. We assume that a bank that passes the stress test is deemed high-type and is allowed to operate at χ^o.[23] A failed bank is deemed to be of the low-type, and the regulator strives to align the capital-ratio requirement towards $\chi^o_L \geq \chi^o$ by imposing a surcharge $x \geq 0$.

For banks that pass the test, we do not consider a negative surcharge ie no capital relief. In part, this is motivated via stress-testing in practice.[24] At the same time, given that negative surcharges are subject to the same regulatory issue of potentially misdirected requirements as long as $q_H < 1$, we do not expect negative surcharges to lead to new insights. Nonetheless, in an alternative setup where the baseline requirement is equal

[23]Note that in practice, most jurisdictions no longer assign an official pass or fail grade to the banks. Yet, whether a bank has done well or not in absolute (i.e. pass or fail) terms can still be inferred from its capital ratio under stress and the minimum capital-ratio requirement in the economy.

[24]In the U.S., for instance, banks that perform poorly in the test (in terms of their losses under stress) must satisfy a higher capital surcharge, while those that do well are not given any relief and must continue to satisfy the baseline requirements.

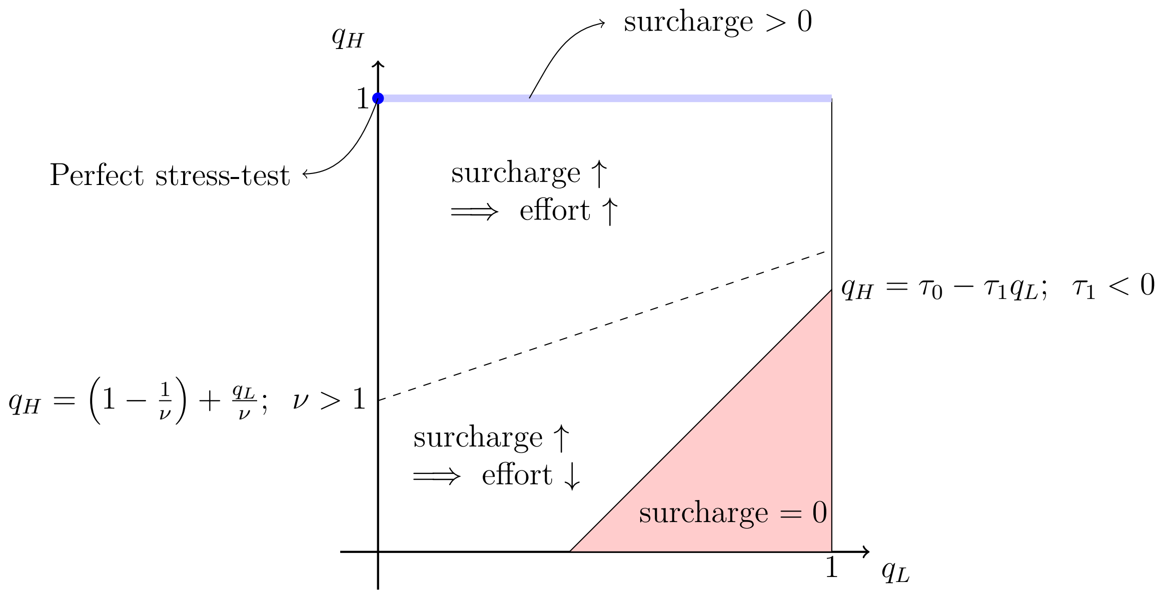

Figure 2: **Stress-test accuracy, effect on ex-ante effort, and optimal penalties:** Each point on the unit square characterises the accuracy of a stress-test. A higher q_H and a lower q_L indicate a more accurate test. The optimal surcharge is zero if accuracy is below the solid diagonal line (i.e. in the shaded area), and positive if $q_H = 1$. Effort increases with the surcharge above the dotted diagonal line, and decreases otherwise.

to that of the high-type bank, the regulator would optimally never choose a negative surcharge, and yet face the same qualitative trade-offs as in our original setup.[25]

The core question of interest then is as follows: what is the welfare maximising level of surcharge x that the regulator must announce on date-0. The choice of x is non-trivial, and is subject to a three-way trade-off.

1. In case of the low-type bank, the surcharge (upon failing the test) **increases** welfare *ceteris paribus* as long as $x \leq \chi_L^o - \chi^o$. This is because the surcharge brings the requirement $(\chi^o + x)$ closer to the optimal (χ_L^o).

2. In case of the high-type bank, the surcharge (upon failing the test) **decreases** welfare *ceteris paribus*. This is because $\chi^o + x > \chi^o \geq \chi_H^o$, as a result of which the surcharge takes the effective requirement away from the optimal.

[25]That is, Lemma 6 and Propositions 2 and 3, which together form the main results of the paper, continue to hold.

3. The surcharge affects the wedge between the expected value of being high- versus low-type on date-1, and thus impacts the bank's behaviour on date-0. Depending on the accuracy of the stress test, this can lead to an increase or decrease in the bank's effort. We prove this result in Lemma 6 below. Accordingly, *ceteris paribus*, a higher surcharge can **increase or decrease** welfare through its effect on effort.

Lemma 6. *The bank's effort may increase or decrease with a surcharge, depending on the accuracy of the stress test.*

Proof. The date-0 problem of the bank is:

$$\max_{e} \quad -\zeta(e) + \beta p(e) \underbrace{\left(q_H V_H(\mathcal{X}^o) + (1-q_H)V_H(\mathcal{X}^o + x)\right)}_{\mathbb{E}V_H} + \beta(1-p(e)) \underbrace{\left(q_L V_L(\mathcal{X}^o) + (1-q_L)V_L(\mathcal{X}^o + x)\right)}_{\mathbb{E}V_L} \tag{15}$$

We begin by noting that similar to the case without stress testing, the effort the bank exerts increases with the *expected* value function wedge $\omega = \mathbb{E}V_H - \mathbb{E}V_L$. Taking the derivative of ω with respect to x at $x = 0$ gives:

$$\left.\frac{\partial \omega}{\partial x}\right|_{x=0} = (1-q_H)V_H'(\mathcal{X}^o) - (1-q_L)V_L'(\mathcal{X}^o)$$

where V' indicates the derivative of the value function. To determine the sign of this expression, divide everything by $V_L'(\mathcal{X}^o)$:[26]

$$sgn\left(\left.\frac{\partial \omega}{\partial x}\right|_{x=0}\right) = -sgn\left((1-q_H)\underbrace{\frac{V_H'(\mathcal{X}^o)}{V_L'(\mathcal{X}^o)}}_{\nu} - (1-q_L)\right)$$

Next, recall from the proof of Lemma (5) that $V_H'(\mathcal{X}^o) - V_L'(\mathcal{X}^o) < 0$, which implies that

[26] Since the value of a more regulated bank is lower, $V_L'(\mathcal{X}^o) < 0$. As such, we add a minus sign to the RHS expression.

$\nu > 1$ since $V'_H(\mathcal{X}^o) < 0$ and $V'_L(\mathcal{X}^o) < 0$. Thus, the effect of surcharge on the bank's effort choice depends on the accuracy of the test as follows:

$$(1 - q_L) - (1 - q_H)\nu \begin{cases} > 0 & \implies \quad \text{efforts increases with surcharge} \\ = 0 & \implies \quad \text{efforts does not change with surcharge} \\ < 0 & \implies \quad \text{effort decreases with surcharge} \end{cases}$$

■

Intuitively, ν captures the relative shadow cost of tightening regulation for the high- and low-type banks. *Ceteris paribus*, a higher ν makes imposing a surcharge less desirable by making it more likely that the bank reduces effort. Similarly, for a given ν, a higher Type-I (i.e. lower q_H) or Type-II error rate (higher q_L) would make $(1-q_L)-(1-q_H)\nu$ more negative and cause the bank to reduce effort following a higher surcharge. Indeed, if a high-type bank is sufficiently likely to fail the stress-test and the low-type bank is sufficiently likely to pass, then the high-type bank will often face a surcharge while the low-type bank will not, thereby reducing the relative benefit of being a high-type bank. This will induce the bank to exert less effort towards becoming high-type in the first place. Relatedly, it is clear from Lemma 6 that with a perfect stress test, i.e. when $(q_H = 1, q_L = 0)$, effort increases with surcharge, while when $q_H = q_L = 0.5$, effort decreases with surcharge. We indicate these insights qualitatively (i.e., not to scale) in Figure 2.[27]

Next we assess the relationship between accuracy of the stress-test and the optimal surcharge.

Proposition 2. *No surcharge must be imposed if the accuracy of stress testing as measured by a (well-defined) linear combination of the Type-1 and Type-II error rates is higher than a cutoff.*

[27] In case the test has non-trivial discriminatory power, the set of parameters of interest is $q_H > q_L$.

Proof. Welfare as a function of the surcharge x can be written based on the regulator's problem as follows (note that e also depends on x in this expression):

$$\max_{x} \quad W(x) = \beta p(e)\Big(q_H U_H(\mathcal{X}^o) + (1-q_H)U_H(\mathcal{X}^o + x)\Big) +$$

$$\beta(1-p(e))\Big(q_L U_L(\mathcal{X}^o) + (1-q_L)U_L(\mathcal{X}^o + x)\Big) - \zeta(e)$$

Our goal is to identify 'a' non-trivial set of (q_H, q_L) where $W(0) > W(x)\ \forall\ x > 0$, i.e. a zero surcharge is optimal.[28] A sufficient condition for this to be the case is $W'(x) < 0\ \forall\ x > 0$. To this end, we consider the first-order condition of the regulator's problem:

$$\frac{dW}{dx} = p'(e)e'(x)\Big(q_H U_H(\mathcal{X}^o) + (1-q_H)U_H(\mathcal{X}^o + x)\Big) + p(e)(1-q_H)U'_H(\mathcal{X}^o + x) -$$

$$p'(e)e'(x)\Big(q_L U_L(\mathcal{X}^o) + (1-q_L)U_L(\mathcal{X}^o + x)\Big) + (1-p(e))(1-q_L)U'_L(\mathcal{X}^o + x) - \zeta'(e)e'(x)$$

To characterise the sign of this expression, we make a few assumptions, again with the goal to find *sufficient* conditions under which the optimal surcharge is zero.

– First we assume that $x \in [0, \mathcal{X}_L^o - \mathcal{X}^o]$. The upper bound corresponds to a surcharge amount that results in a requirement for the low-type banks that is equal to the ex-post optimal requirement $\mathcal{X}_L^o$. In principle, the optimal surcharge could be higher (due to its effect on improving ex-ante effort), but that would entail a welfare decreasing effect in case of both high- and low-type banks.

– Second, we assume that (q_H, q_L) are such that the effort exerted by the bank decreases as surcharge increases (as per Lemma 6).

Next, since $U_s(\mathcal{X}^o + x), s \in \{L, H\}$ is a concave function of x, $\mathcal{X}_L^o \geq \mathcal{X}^o \geq \mathcal{X}_H^o$ implies the following: (i) $U_H(\mathcal{X}^o) \geq U_H(\mathcal{X}^o + x)$; (ii) $U'_H(\mathcal{X}^o + x) \leq 0$; (iii) $U_L(\mathcal{X}^o) \leq U_L(\mathcal{X}^o + x)$; and

[28] Our goal is to not fully characterise the set of (q_H, q_L) for which the optimal surcharge is zero. We only wish to show that with low-enough accuracy, imposing a surcharge is sub-optimal.

(iv) $U'_L(\mathcal{X}^o + x) \geq 0$; $\forall\, x \in [0, \mathcal{X}^o_L - \mathcal{X}^o]$. It then follows that:

$$\frac{dW}{dx} \leq p'(e)e'(x)U_H(\mathcal{X}^o) + p(e)(1-q_H)U'_H(\mathcal{X}^o + x) - p'(e)e'(x)U_L(\mathcal{X}^o) +$$

$$(1-p(e))(1-q_L)U'_L(\mathcal{X}^o) - \zeta'(e)e'(x)$$

Finally, we re-arrange and set the right-hand-side expression to zero:

$$p(e)U'_H(\mathcal{X}^o + x) + (1-p(e))U'_L(\mathcal{X}^o) - p(e)q_H U'_H(\mathcal{X}^o + x) - (1-p(e))q_L U'_L(\mathcal{X}^o) +$$

$$\underbrace{p'(e)e'(x)\Big(U_H(\mathcal{X}^o) - U_L(\mathcal{X}^o)\Big)}_{A<0} - \zeta'(e)e'(x) = 0$$

$$\implies \underbrace{\frac{A}{p(e)U'_H(\mathcal{X}^o + x)} + 1 + \frac{(1-p(e))U'_L(\mathcal{X}^o)}{p(e)U'_H(\mathcal{X}^o + x)} - \zeta'(e)e'(x)}_{\tau_0 <> 0} - q_L \underbrace{\frac{(1-p(e))U'_L(\mathcal{X}^o)}{p(e)U'_H(\mathcal{X}^o + x)}}_{\tau_1 < 0} = q_H$$

$$\implies q_H = \tau_0 - \tau_1 q_L \tag{16}$$

In equation (16), while the slope is positive, the intercept can be positive or negative, depending on the underlying parameters. The equation implies that when $q_H < \tau_0 - \tau_1 q_L$ the surcharge should be zero, as also indicated in Figure 2. ■

Intuitively, the proposition shows that when q_H is low and/or q_L is high – both of which reflect a relatively less accurate stress-test – the surcharge must be zero. Next we explore conditions under which the optimal surcharge can be strictly positive.

Consider a stress-test that is accurate in identifying high-type banks i.e. $q_H = 1$, but is possibly inaccurate in identifying low-type banks i.e. $1 > q_L \geq 0$. In this case a higher x does not affect $\mathbb{E}V_H$, but decreases $\mathbb{E}V_L$ (recall equation (15)). As a result, the banker increases effort as surcharge increases. Second, consider the regulator's problem:

$$\max_x \quad \beta p(e)U_H(\mathcal{X}^o) + \beta(1-p(e))\Big(q_L U_L(\mathcal{X}^o) + (1-q_L)U_L(\mathcal{X}^o + x)\Big) - \zeta(e)$$

A higher x does not affect welfare when the bank passes the test, but increases welfare when it fails the test as long as $x \leq \chi_L^o - \chi^o$ (recall from Proposition 4 that beyond this threshold, the effective requirement on the low-type bank is higher than the optimal requirement χ_L^o.)

Combining the effect of a surcharge on effort e and $U_L(\chi^o + x)$, both of which increase as x increases, and given that $U_H(\chi^o) > U_L(\chi^o)$, it is clear that welfare, ignoring the effect of the surcharge on the cost of effort, must increase as x rises above zero. Thus, if the cost of effort is sufficiently small, the optimal surcharge must be strictly positive. Together with proposition 2, this insight points to a material shift in the relation between optimal surcharge and stress-test accuracy, with the optimal surcharge being zero (positive) if the level of accuracy of the stress tests is sufficiently low (high).

In what follows, we formalise this insight using a simpler version of the model where the probability that a bank is of a given type is fixed.[29]

Proposition 3. *The optimal surcharge increases with stress-test accuracy.*

Proof. The regulator's problem in this case is as follows:

$$\max_x \quad \beta p\Big(q_H U_H(\chi^o) + (1-q_H)U_H(\chi^o + x)\Big) + \beta(1-p)\Big(q_L U_L(\chi^o) + (1-q_L)U_L(\chi^o + x)\Big)$$

The first order condition is:

$$[x] \quad 0 = p(1-q_H)U_H'(\chi^o + x) + (1-p)(1-q_L)U_L'(\chi^o + x)$$

Next consider an increase in accuracy via a higher q_H (the proof in case of a lower q_L is similar):

$$0 = -pU_H'(\chi^o + x) + p(1-q_H)U_H''(\chi^o + x)\frac{\partial x}{\partial q_H} + (1-p)(1-q_L)U_L''(\chi^o + x)\frac{\partial x}{\partial q_H}$$

[29]A similar result cannot be proven analytically in the fully specified model. Although numerical simulations show that the result also holds in the fully specified model.

Since U is concave, and $U'_H(\mathcal{X}^o + x)$ is negative (because $\mathcal{X}^o$ is higher than the optimal requirement for the high-type bank), $\frac{\partial x}{\partial q_H} > 0$. ■

3.5 Endogenous accuracy

Thus far, we have considered the accuracy of the stress test – as summarised by (q_H, q_L) – to be given exogenously. In reality, regulators may be able to influence or even choose the level of accuracy, and may prefer to increase it given the welfare gains it entails. Yet, they may be constrained by various factors. In this section, we discuss two cases summarizing the trade-offs regulators face in choosing a higher level of test accuracy.

Case I: Improving accuracy along one or both dimensions possible Consider the case where the regulator is able to improve accuracy along one or both dimensions i.e. increase area under the receiver operating characteristic (ROC) curve (recall discussion in Section 3.4). This could be achieved, for instance, by making the test harder – e.g. by using a more severe crisis scenario – to lower the false negative rate, and at the same time exercising greater caution in assessing test results to mitigate any increase in the false positive rate as a result of a harder test. Designing such a test is likely to be more costly not just for the regulator, but also for the banks. Indeed, a more extensive review of the banks' balance sheet and its risk models would not only absorb additional supervisory force, but also more bank resources.

To model these trade-offs, we consider the problem of a regulator that jointly chooses surcharge x and a test-design parameter $y \geq 0$ that maps to the pass probability of the high-type bank: $q_H(y) \uparrow 1$ as $y \to \infty$, while keeping q_L fixed.[30] In addition, we assume that adjusting the design of the test y to improve accuracy entails a social cost $C(y) = \gamma_c y$. This setup leads to the following result, which we prove in Appendix B.

[30] This is without loss of generality: the other case where q_L is adjusted can be handled similarly and leads to similar conclusions.

Proposition 4. *The regulator increases stress-test accuracy q_H as well as the surcharge for failing banks as the cost of accuracy decreases.*

Intuitively, higher accuracy along one or both dimensions of the stress-test reduces the likelihood that a high-type bank is penalised, and this in turn mitigates the adverse incentives that a capital surcharge can generate. As such, a higher surcharge is optimal.

Case II: Improving accuracy along both dimensions not possible In practice, in contrast to the previous case, it may not be possible to improve accuracy along one dimension without worsening it along the other dimension. For instance, a prohibitively high cost of making the test more comprehensive may leave the regulator in a situation where reducing one error rate invariably increases the other error rate. Or there may be fundamental constraints to improving accuracy given that predicting bank performance in a hypothetical scenario rests on a number of assumptions, and is an inherently hard endeavor.[31] In practical terms, this situation implies that the regulator cannot increase the area under the ROC curve, and can only move *along* it, i.e. vary the signal cutoff for failures.

To obtain the regulatory implications in this case, we assume without loss of generality that by reducing false positive rate (i.e. increasing q_H), the regulator also ends up increasing the false negative rate (i.e. higher q_L). In this case, we find that a lower cost of improving accuracy along the q_H dimension induces the regulator to improve the accuracy along that dimension but does not necessarily enable the bank to choose a higher x, thus creating a dichotomy between accuracy and surcharge (see proof in Appendix C).

Overall, our analysis suggests that stress-test design and the subsequent capital surcharge decisions are intricately linked, and must inform each other. This is especially given our finding that higher assessment accuracy along one dimension does not necessarily imply room to impose a higher surcharge on banks.

[31] See Parlatore and Philippon [2020] for a discussion of the underlying technical constraints.

3.6 Additional policy trade-offs

Disclosure policy A contrasting aspect of stress-testing compared to other forms of micro-prudential supervision and regulation is that the testing methodology and test results are disclosed to the wider public in quite some detail. Disclosure of results can have an additional impact on banks via market discipline, for instance via the surprise element in test results, i.e. the difference between how investors perceive a bank and its stress-test performance. Depending on the direction of surprise in test results, investors may seek a higher or lower return for providing funding to banks. This can impact how banks respond to stress-tests, and have implications not only for stress-test disclosure policy, as discussed in [Goldstein and Sapra, 2014; Goldstein and Leitner, 2018; Leitner and Williams, 2020], but also for how test-based capital requirements must be set.[32]

To assess this latter aspect, we extend our model to include a role for uninsured investors that react to stress-test results. To create an incentive for the bank to pursue the two types of funding, we assume that deposit based funding is not easily scalable, and thus the unit cost of deposit funding $R(d)$ increases with the funding amount. At the same time, investor funding w, even though more costly for smaller amounts, is easily scalable, and is the relatively cheaper source of financing for larger amounts (see Figure 3). Yet, when a bank fails the stress-test, while insured depositors do not seek a higher return, uninsured investors raise their required return $Q(w)$ by, say, δ.[33] The date-1 problem of the bank in

[32]Other studies in this literature include Corona et al. [2019] who assess how bailout regime and disclosure policy interact, Orlov et al. [2018] who characterise the optimal disclosure policy for high- and low-risk banks, and Bouvard et al. [2015] who show that the optimal disclosure policy must vary along the business cycle.

[33]Relatedly, Chen et al. [2020] provide empirical evidence of the fact that uninsured deposit flows are more sensitive to information about bank performance.

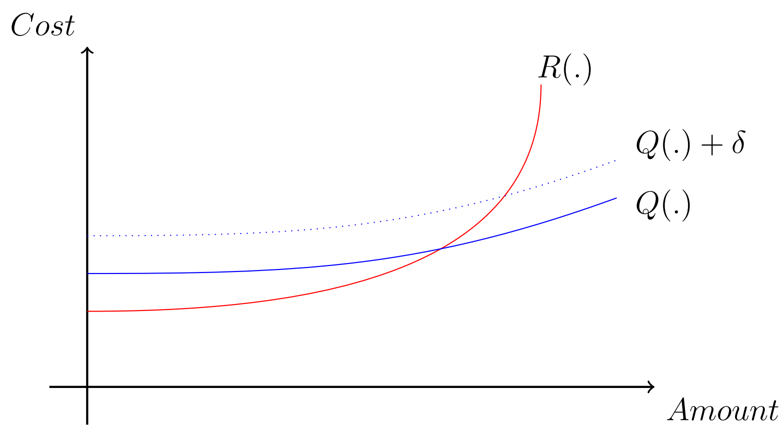

Figure 3: Cost of insured deposits and uninsured investor funding

this case is as follows:

$$V_s(\chi_s) = \max_{d,w} \quad \beta \int_{\frac{R(d)d+Q(w)w}{g(k+d+w)}}^{\infty} \Big(\psi g(k+d+w) - R(d)d - Q(w)w\Big) f_s(\psi) d\psi$$
$$s.t. \quad \frac{k}{\chi_s} \geq (d+w). \tag{17}$$

Assuming that both forms of financing are used in equilibrium, we assess the implications for banks and for the regulator. We first note that failure in the test is now more costly for the bank – not only does it need to satisfy a higher capital ratio, its unit cost of funding is higher compared to the case where disclosures have no material impact (i.e. $\delta = 0$). Formally, the FOCs of the bank's problem imply that d and w are determined in the case of passing and failing banks as follows, respectively:

$$\frac{k}{\chi} = d + w; \quad R'(d)d + R(d) = Q'(w)w + Q(w)$$

$$\frac{k}{\chi + x} = d + w; \quad R'(d)d + R(d) = Q'(w)w + Q(w) + \delta$$

To make analytical progress, we assume simple forms of the cost functions: $R(d) = R_0 + R_1 d$ and $Q(w) = Q$ such that they continue to reflect the underlying intuition that investor funding is more elastic than deposit funding. Solving the FOCs explicitly leads to:

$$d_{pass} = \frac{Q - R_0}{2R_1}; \quad w_{pass} = \frac{k}{\chi} - \frac{Q - R_0}{2R_1};$$

$$d_{fail} = \frac{Q + \delta - R_0}{2R_1}; \quad w_{fail} = \frac{k}{\chi + x} - \frac{Q + \delta - R_0}{2R_1};$$

That is, upon failure in the test, the bank reduces its overall balance sheet and funding, and tilts its funding composition towards deposits. At the same time, the total funding cost (TC) of a failing bank is increasing in δ.[34] In turn, the value of a failing bank is decreasing in δ.

To derive the implications for the effort the bank exerts on date-0, we assess the impact of δ on the expected value function wedge (recall equation 6). The value of a high- or low-type bank that passes the test – ie $V_H(\chi^o)$ and $V_L(\chi^o)$ – remains unaffected by δ. However, δ leads to a larger decline in the value of a high-type bank that fails the test. To see this formally, consider the resolved value function of the s-type bank, where we have already solved for the d, w decisions as a function of δ:

$$V_s(\chi^o + x; \delta) = \beta \int_{\frac{TC(\delta)}{g(k+d+w)}}^{\infty} \Big(\psi g(k + d + w) - TC(\delta)\Big) f_s(\psi) d\psi$$

The derivative of the value function with respect to δ implies:[35]

$$\frac{dV_s(\chi^o + x; \delta)}{d\delta} = -\beta TC'(\delta) \int_{\frac{TC(\delta)}{g(k+d+w)}}^{\infty} f_s(\psi) d\psi$$

The above expression proves that the value function of each type of bank is decreasing in δ since $TC'(\delta) > 0$. Moreover, since the failure cutoff and $TC(.)$ are independent of

[34] To see this, consider the total funding cost (TC) of a failing bank as a function of $\delta : TC(\delta) = R(d)d + (Q + \delta)(k/(\chi + x) - d)$ where $d = (Q + \delta - R_0)/(2R_1)$. Taking the derivative of the above expression with respect to δ immediately leads to the above result: $TC'(\delta) > 0$.

[35] Note that $d + w$ is independent of δ and that only one of the terms following an application of the Leibniz rule is non-zero.

bank type, the decline in value is greater in case of the high-type bank. Intuitively, the probability that the high-type bank is solvent is higher, which means that it is more likely to incur the higher funding cost. Therefore, it follows that $V_H(\mathcal{X}^o+x;\delta=0)-V_H(\mathcal{X}^o+x;\delta>0) > V_L(\mathcal{X}^o+x;\delta=0) - V_L(\mathcal{X}^o+x;\delta>0)$. As such, *ceteris paribus,* a higher δ depresses the expected value function wedge.

The above analysis shows that depending on the accuracy of stress-tests, disclosing the results of the test can strengthen or worsen the bank's ex-ante incentives. When the test is sufficiently accurate, the disclosure can help improve market discipline and increase the effort banks' exert ex-ante. Yet, when the test is less accurate (ie moving south-east from the north-west corner in Figure 2), disclosure can worsen ex-ante incentives, and place further constraints on using stress-test results for imposing bank-specific capital surcharges.

Failure costs Failure of a bank can impose a social cost. This cost can stem from, for instance, forced sale of a failed bank's assets, as well as due to resolution related expenses. It can be a major cost in the case of large banks (due to contagion/knock-on effects), when the resolution framework is not well functioning, or during a crisis when many banks are in insolvency at the same time.

Failure costs exacerbate the trade-off for assessment based regulation. A higher surcharge (compared to the case without failure costs) may be justified on the grounds that it lowers the expected failure rate and attendant social costs. Yet, to the extent assessment is not sufficiently accurate, a higher surcharge in the case of a high type bank would not only lower welfare, but also would lower the ex-ante effort exerted by the bank. As such, it is not obvious as to whether the surcharge must be adjusted upwards or downwards in the presence of higher failure costs.

To formally assess the effect of failure cost on optimal regulation, we adapt the model as follows. We assume that once a bank fails, the recovery value of its assets is less that a hundred percent. This cost – denoted Δ – is borne by the deposit insurance program and is funded via taxes:

$$T(\psi) = \begin{cases} Rd - \psi g(k+d)(1-\Delta) & \text{If the bank fails i.e. } \psi \leq \frac{Rd}{g(k+d)} \\ 0 & \text{Otherwise} \end{cases}$$

In what follows, we prove that the failure cost exacerbates the inefficiency banks pose, and rationalises a higher ex-post requirement $\mathcal{X}^o$ and also a higher ex-ante surcharge x associated with failing the stress test.[36]

We begin by assessing the ex-post requirement, while abstracting away from bank-type as before. Household and banker consumption on date-2 in this case is given as:

$$c_2 + n = \bar{Y} + \psi g(k+d) - \Delta \psi g(k+d) \mathbb{1}\left(\psi \leq \frac{Rd}{g(k+d)}\right).$$

Accordingly, the planner's problem is:

$$\max_{d} \quad (1+\beta)\bar{Y} + \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g(k+d) - Rd\Big) df(\psi) + \beta \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g(k+d) - Rd - \Delta \psi g(k+d)\Big) df(\psi),$$

while the attendant first-order-condition is:

$$\begin{aligned} 0 \;=\; & \beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi \; + \\ & \underbrace{\beta \int_{0}^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d)(1-\Delta) - R\Big) f(\psi) d\psi - \beta \Delta \psi g(k+d) \frac{\partial \frac{Rd}{g(k+d)}}{\partial d} f\left(\frac{Rd}{g(k+d)}\right)}_{\text{Bank-failure inefficiency}} \end{aligned} \tag{18}$$

We know from the discussion of equation (11) that the inefficiency term in that equation, namely $\beta \int_0^{\frac{Rd}{g(k+d)}} \Big(\psi g'(k+d) - R\Big) f(\psi) d\psi$, is negative. This means that the left term in the second row of equation (18) is also negative, and even lower in value. At the same time,

[36]We assume that the cost of failure is fixed. In practice, implicit guarantees for too-big-to-fail banks may imply that Δ is smaller for larger banks, which in turn can induce banks to pursue leverage. Yet, we abstract away from such considerations to keep the model focused.

since $g(.)$ is concave:

$$\frac{\partial \frac{Rd}{g(k+d)}}{\partial d} = \frac{R\Big(g(k+d) - dg'(k+d)\Big)}{g(k+d)^2} > 0.$$

As such, the inefficiency term in equation (18) is negative and larger in magnitude relative to the inefficiency term in equation (11). Thus failure cost amplifies the bank-failure inefficiency. In turn, as shown in Lemma 3, greater inefficiency rationalises a higher minimum capital-ratio requirement. We note this result in Lemma 7.

Lemma 7. *The regulator must optimally impose a higher ex-post minimum capital-ratio requirement on a bank that, all else equal, exhibits a higher failure cost.*

Next we examine how the optimal surcharge must change as failure cost increases. Unfortunately, it is not possible to characterise the change generally. For analytical tractability, we assume (as in the previous subsection) that the probability of being a high-type (or equivalently low-type) bank is given and that there is no effort choice involved. We find that irrespective of the accuracy of the test (and thus the attendant adverse incentives it generates), the optimal surcharge is higher when failure is more costly (see proposition below and proof in Appendix D).

Proposition 5. *Assuming $p(e) \equiv p$, the optimal surcharge must increase as Δ increases.*

Relaxing the assumption that $p(e) = p$ does not lead to a general result, that is, $\frac{dx}{d\Delta}$ cannot be signed unless the specific values of the parameters of the model are known. As such, we pursue this more general case in the quantitative analysis. Nonetheless, the above proposition suggests that if the stress test is sufficiently accurate so that effort e and thus the probability of being a high-type bank increase as the surcharge increases, then it is likely that the surcharge must be optimally adjusted upwards as the failure cost increases.

Parameter	Description	Value	Target moments	Value
α	Payoff exponent: $(\mathrm{k+d})^{\alpha}$	0.914	Gross Return on risk-adjusted assets	10.19%
μ	Mean of ψ	1.336	Equity capital to assets ratio	10.38%
σ	Standard-deviation of ψ	0.102	Value-at-risk threshold	1%
$\bar{Y}$	Household income	117.8	Household savings rate	7.32%
β	Discount factor	0.99	Deposit interest-rate	1%
Δ	Failure cost	0.22	US bank failure losses	22%

Table 1: Parameter values and target moments. Bank micro-data are sourced from Fitch, US household savings rate from FRED, and bank failure losses from FDIC. Note that the last two parameters and target moments have a one-to-one mapping (i.e. they need not be estimated jointly), and that without loss of generality k is normalised to unity. The value of the moments in data are exactly match with those implied by the mode.

4 Numerical illustration

We now calibrate the model parameters using data on U.S. commercial banks. Our goal is not to pursue a quantitative analysis of the model or draw empirical predictions, but provide a relevant numerical illustration of our analytical results. To this end, we set the parameters such that model generated moments are equal to the corresponding data moments (see Table 1). We focus on the post-GFC to pre-Covid period – i.e. 2010-2019 – to abstract away from any crisis led aberrations in the data.

We consider the following moments as targets. First is the pooled mean of return on risk-weighted assets, while taking into account interest as well as non-interest income. Dividing by risk-weighted assets (instead of just assets) helps align the moment condition with the interpretation of high- and low-type bank in our model (recall that high- and low-type banks have the same standard deviation of return on assets, and vary only in terms of the mean return on assets). Second is the pooled mean of equity capital to assets ratio. Third is a typical regulatory or bank-management imposed value-at-risk threshold of 1%. Fourth is the household savings rate, defined as the average savings of US households out of their personal disposable income during 2010-2019. Next, we set the interest rate to 1% – a standard value in the literature. Finally, Δ is set in line with the losses associated with bank failures in the US during 2010-2019. According to the Federal Deposit Insurance Commission (FDIC), there have been 367 bank failures during this period, and the median estimated loss is about 21% of the failed bank's assets, while the attendant inter-quartile range is 13% to 30%. Our target moment is the mean, which is 22%.

As regards the functional forms, we assume the cost of exerting effort by the bank on date-0 as $\zeta(e) = \gamma_e e^2$, and the attendant probability of the bank becoming a high-type on date-1 as $p(e) = 1 - 1/(1+e)$. The exact functional forms do not matter for our qualitative results as long as $\zeta(.)$ is (weakly) convex and $p(.)$ is concave. We choose γ_e so that the bank is high- or low-type with equal probability. As regards μ_H and μ_L, we assume a symmetric perturbation of 50 basis points around μ. Finally, we treat q_H and q_L as free parameters that we conduct comparative statics with respect to.

Optimal ex-post regulation We begin by analyzing the impact of a minimum capital-ratio requirement on the bank's behavior and overall welfare on date-1. Without loss of generality, we consider a high-type bank. Starting from the unregulated economy, a higher minimum capital-ratio requirement forces the bank to deleverage (first panel in Figure 4). This reduces the failure probability (second panel), but also lowers expected output (third panel). The overall effect – one that weighs welfare gains from lower bank failure against the welfare loss from lower expected output – is an inverted U-shaped welfare profile as a function of χ. This finding is consistent with Lemmas 2 and 3 where we showed that the unregulated equilibrium is sub-optimal and that a minimum capital-ratio requirement can improve welfare, and also with the broader literature (e.g. Begenau [2020], Christiano and Ikeda [2016]).

Relatedly, as bank failure costs increase, not only is the optimal requirement higher (as proven in Lemma 7), the welfare gain from regulation is also higher (see left-hand panel in Figure 5).

Finally, we compare the optimal ex-post requirement for low- and high-type banks. Consistent with Lemma 4, we find that the requirement is higher for the low-type bank (see right-hand panel of Figure 5, dotted lines).

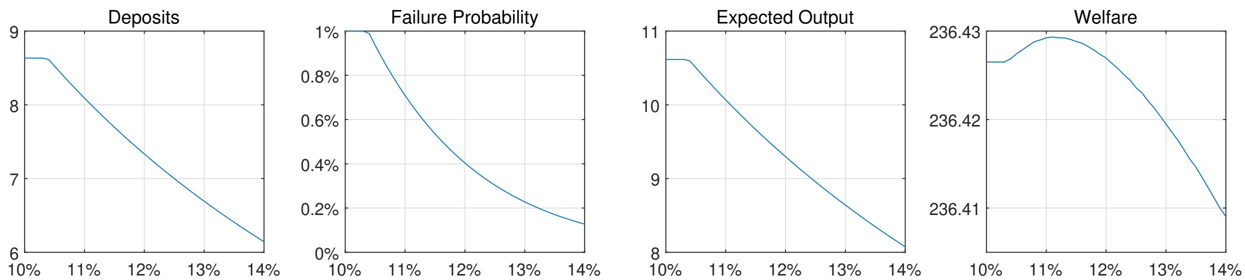

Figure 4: The effect of minimum capital-ratio requirement (x-axis) on the high-type bank and on overall welfare.

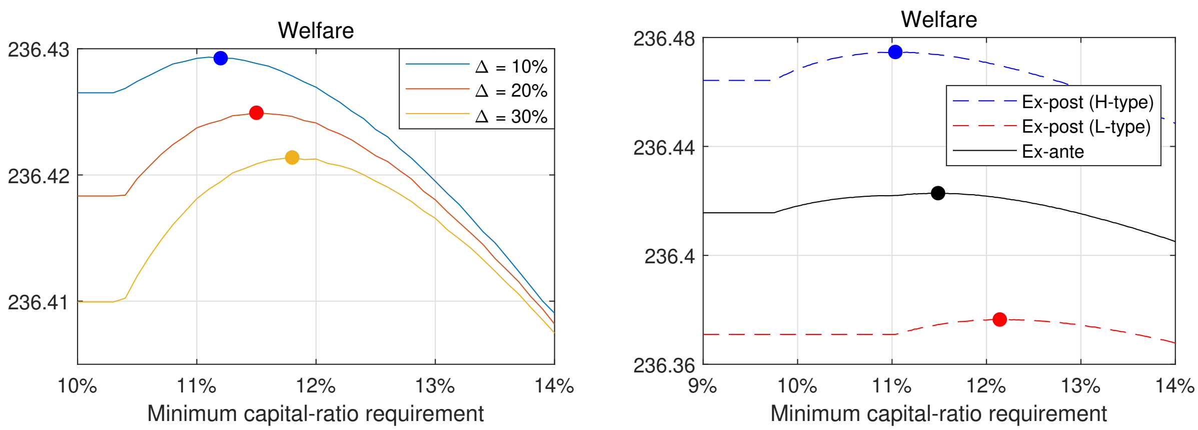

Figure 5: *Left-hand panel*: The welfare maximizing regulation for varying levels of bank failure costs. *Right-hand panel*: Optimal ex-post requirement depending on bank type, and the optimal ex-ante requirement in the absence of stress tests.

Optimal ex-ante regulation When the regulator cannot observe banks' types ex-post, the optimal ex-ante requirement announced on date-0 cannot be bank-type specific. Consistent with Proposition 1, we find that it is saddled by the ex-post optimal requirements (see solid line in the right-hand panel of Figure 5).

Next we assess how a stress-test led surcharge affects bank's behavior. A higher surcharge decreases the value of both high- and low-type banks (left-hand panel of Figure 6). The decrease is starker for a high-type bank – indeed the opportunity cost of not being able to use its balance sheet capacity is higher for a bank whose assets have a higher return. And as long as the stress test is not fully perfect, both $\mathbb{E}V_H$ and $\mathbb{E}V_L$ decrease as x increases.

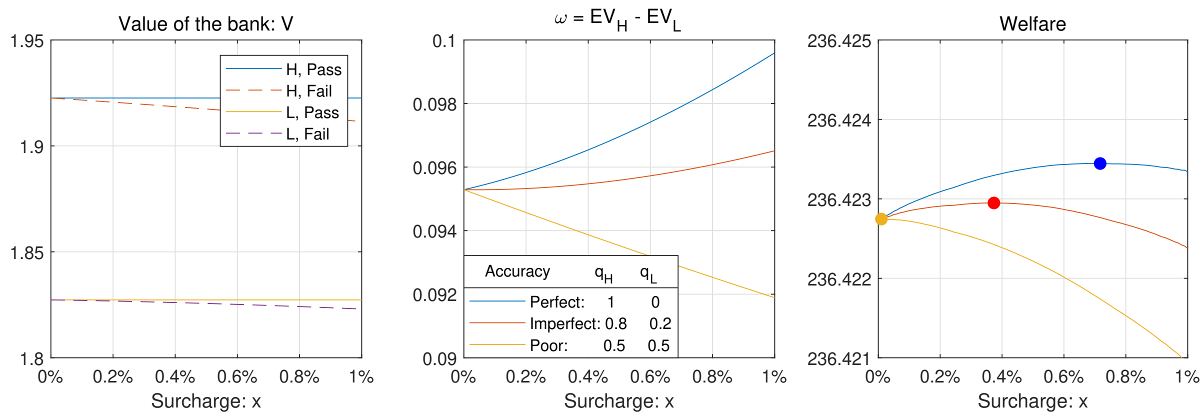

Figure 6: *Left-hand panel:* The value V of the bank in various cases as a function of the surcharge. *Centre panel:* The expected value wedge changes in response to the surcharge for different levels of accuracy of the stress test. *Right-hand panel:* Optimal surcharge.

The difference between $\mathbb{E}V_H$ and $\mathbb{E}V_L$, namely ω – as we showed in Lemma 6 – can increase or decrease depending on the accuracy of the test (see centre panel of Figure 6). This immediately means that the effort banks exert can also increase or decrease as the surcharge is raised (recall that e depends on ω; see the proof of Lemma 5). This is a key insight of the paper – a higher surcharge may not necessarily act as a disciplining device if the basis on which the surcharge is imposed is not sufficiently accurate.

Overall, the optimal surcharge depends on the following trade-off. Penalizing banks that fail the stress tests can improve welfare to the extent a low-type bank is penalised. As such, a sufficiently inaccurate test may not increase expected welfare. Moreover, in this case, banks may reduce the effort they exert. We confirm this insight quantitatively. For very low level of accuracy, consistent with proposition 2, the optimal surcharge is zero (right-hand panel of Figure 6). For higher levels of accuracy, including the case of a perfect stress test, the optimal surcharge is higher (recall Proposition 3).

We illustrate the optimal surcharge for each accuracy level of the stress-test in the left-hand panel of Figure 7, thus confirming the broad indications sketched in Figure 2. Indeed, a phase shift is evident: for sufficiently low levels of accuracy, the optimal surcharge is zero. Moving closer to a perfect stress test ($q_H = 1, q_L = 0$) increases the size of the optimal

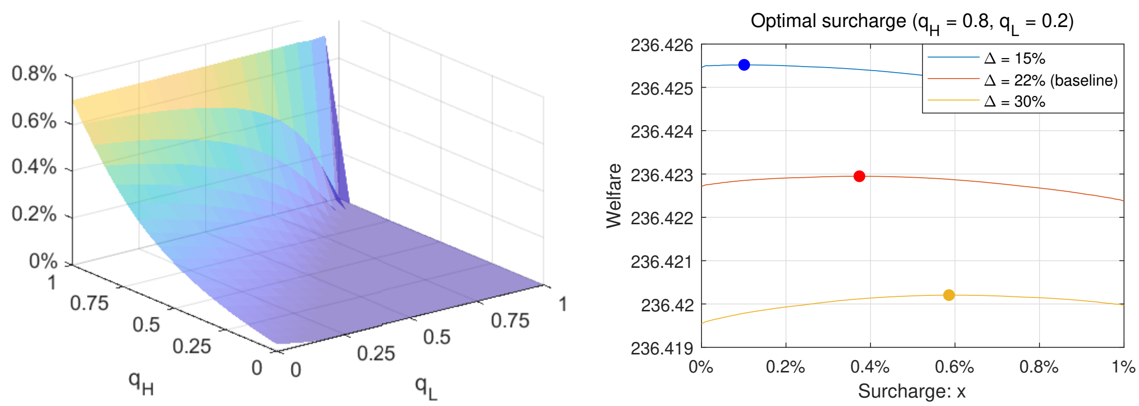

Figure 7: *Left-hand panel*: Optimal surcharge as a function of the accuracy of the stress-test. *Right-hand panel*: Change in optimal surcharge as the cost of failure increases.

surcharge.

Next we elaborate upon the result proven in Proposition 5, and show that as the failure cost increases, the optimal surcharge also increases (see the right-hand panel of Figure 7).

Finally we illustrate the implications of endogensing accuracy (recall the setup in subsection 3.5). We assume that $C(y) = \gamma_c y$, $q_H(y) = 1 - \gamma_q/(1+y)$, and $q_L(y) = 1 - q_H(y)$ (see left-hand panel of Figure 8 for an example). Consistent with the analytical result in proposition 4, we find that as the cost of accuracy decreases ($\gamma_c \downarrow$), the regulator must optimally work with more accurate stress tests ($y \uparrow$), and at the same time, revise upwards the surcharge it imposes on failings banks ($x \uparrow$) (see right-hand panel of Figure 8).

5 The Covid-19 crisis: A test of stress-tests?

In this section, we explore whether the Covid-19 crisis can shed light on how accurately stress-tests identify the riskier banks.

Stress-tests evaluate whether banks have sufficient capital to absorb losses resulting from adverse economic conditions.[37] In the U.S., the Federal Reserve imposes a capital surcharge on banks based on their performance in the test, one aspect of which is the

[37] This involves projecting revenues, expenses, losses, and, crucially, the capital ratios of the participating banks in a recession. The projections use a standard set of capital action assumptions.

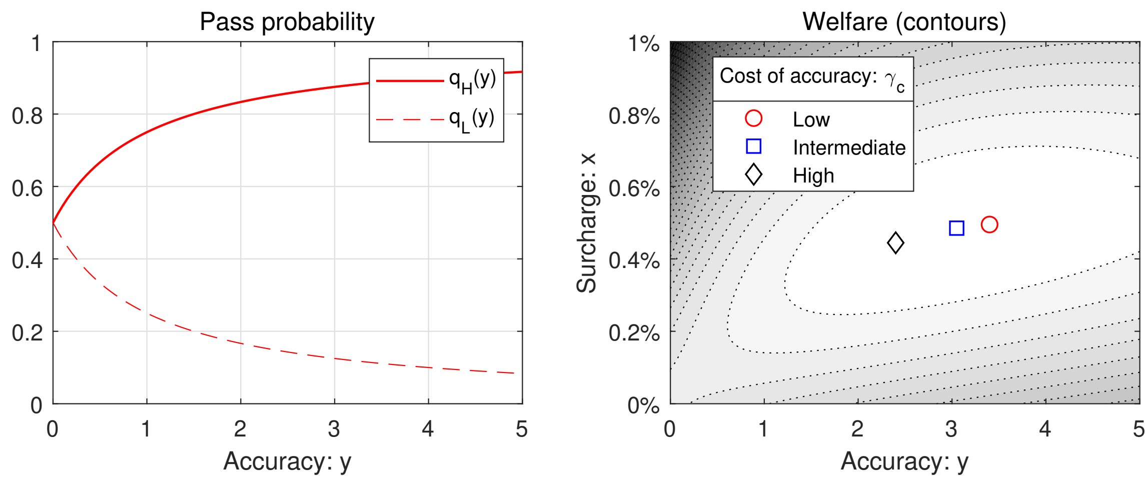

Figure 8: *Left-hand panel:* Pass probabilities for high and low-type banks as a function of accuracy. *Right-hand panel:* The jointly optimal accuracy and surcharge for different levels of cost of accuracy γ_c. Welfare contours correspond to the intermediate level of γ_c.

projected decline in their Common Equity Tier 1 (CET1) capital ratios in the hypothetical severely adverse scenario. Banks that perform poorly face a higher Stressed Capital Buffer (SCB). Figure 9 documents this relationship in case of the stress-test conducted by the Federal Reserve right before the Covid-19 crisis. We can see that beyond a minimum capital surcharge of 2.5%, there is a positive correlation between banks' performance in the test (measured by the decline in their CET1 ratios in the test) and the bank-specific SCB. This shows that a bank's performance in the test has material implications for its regulatory burden.

We compare banks' performance in the 2020 U.S. stress-test with their actual performance in the Covid-19 crisis to learn about potential inaccuracies in stress testing. In what follows, we acknowledge the potential limitations of this exercise, but also discuss reasons for why it can be insightful.

An ideal appraisal of stress-testing would be one where the hypothetical stress scenario and the realised crisis are *identical*. Having a setting like this, however, is close to impossible. Nonetheless, several of the key indicators that characterise a macroeconomic scenario (such as GDP, employment, and stock prices) experienced comparable declines in the 2020

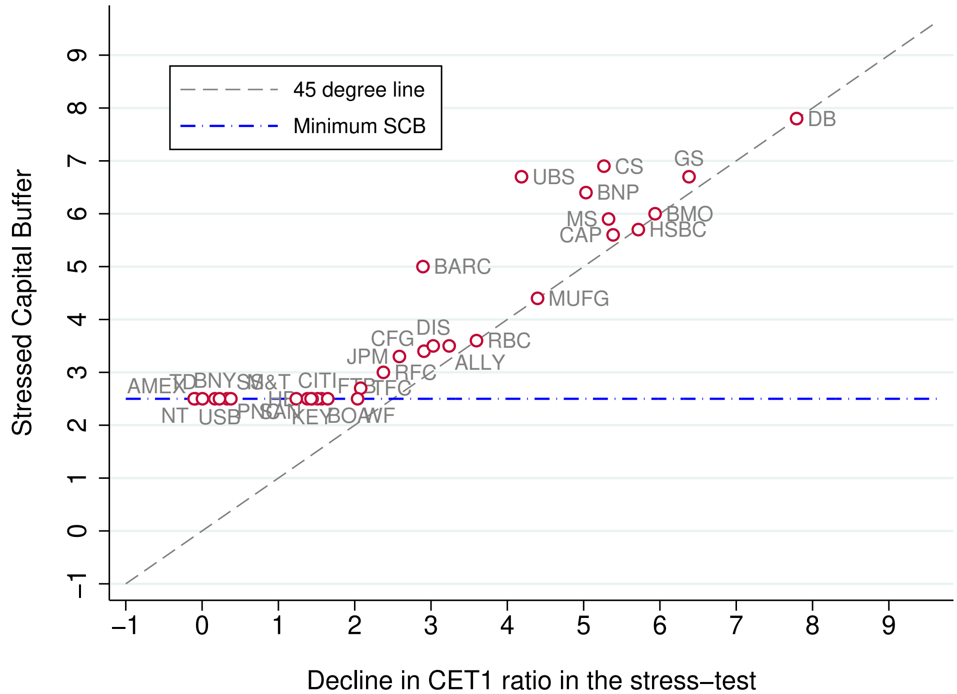

Figure 9: A comparison of the decline in CET1 ratio in the 2020 DFAST and the Stressed Capital Buffer (SCB) imposed on banks. Unit of both axes is percentage points.

US stress-test scenario and the Covid-19 crisis.[38] Given the largely similar deterioration of some major macroeconomic indicators, broad concordance in banks' relative performances in the test and in the current crisis is to be expected (Acharya et al. [2014]).

The Covid-19 shock was completely unexpected, similar to stress-tests where the hypothetical scenarios are not known to banks in advance. This further makes the crisis useful to evaluate stress-tests. Also, given the fact that risk-weighted assets and loan loss provisions (LLPs) are forward looking, and that banks front-loaded their response to the crisis by increasing LLPs substantially in the first half 2020, means that the attendant capital ratios likely reflect how banks would eventually perform in the crisis. This helps address, at least partially, claims that the CET1 ratio of banks may not have bottomed

[38]The hypothetical scenario in the 2020 stress-test in the U.S. comprised of a peak unemployment rate of 10 percent, a decline in real GDP of 8.5 percent, and a drop in equity prices of 50 percent through the end of 2020, among other macroeconomic developments. The U.S. economy contracted by close to 30% (YoY) in Q2 2020; the peak unemployment rate was 15%; and the Dow Jones Index plunged by close to 30% in March 2020.

out yet. Although concerns that extraordinary support was provided to banks and to the broader economy have dented the impact of the crisis on banks (so far) may still apply.

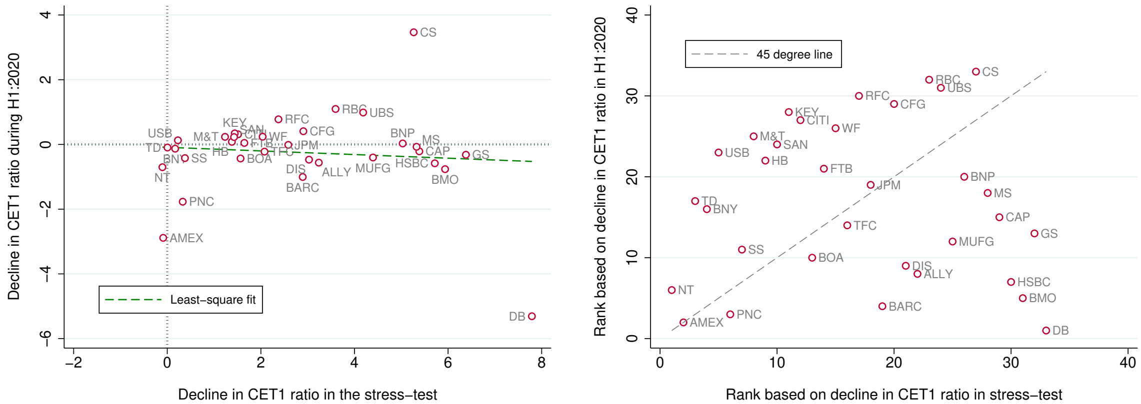

Figure 10: Left-hand panel: A comparison of the decline in CET1 ratio in the stress-test and the actual decline observed between Q4:2019 and Q2:2020. Unit of both axes is percentage points. Right-hand panel: A comparison of the rank based on decline in CET1 ratio in the stress-test and rank based on the actual decline observed between Q4:2019 and Q2:2020. A lower rank (number) indicates a smaller decline in the ratio.

All in all, our assessment is that the Covid-19 crisis is a feasible, if not ideal, test of stress-testing. To this end we compare the test-implied and Q2 2020 CET1 ratios of banks (see Figure 10).[39] We find that the cross-sectional variance in expected changes in CET1 ratios (based on the stress test) is much higher than the observed changes, and that the two do not correlate. In fact, while the ratios declined for almost all banks in the test, it rose for many in reality. In the case of Deutsche Bank USA, for instance, the CET1 ratio declined by 8 percentage points (pp) in the test, while during H1 2020, the same ratio rose by 5 pp. We observe similar degree of discordance in the cross-sectional ranking of banks' CET1 ratios, changes in CDS spreads during the crisis – a market indicator of banks' resilience – and loan loss provisions – an indicator of banks' self assessment of expected credit losses (see Figure 11).[40]

[39] Our conclusions are robust to using Q3 or Q4 2020 data.

[40] Credit losses are a major component of the CET1 ratio projections in the stress-test. This makes loan loss provisions during the Covid-19 crisis a useful indicator to compare banks' stress-test performance with.

The higher variance in test results or its lack of correlation with observed outcomes may not be taken as definitive evidence of noisy assessment. Yet, they raise the question whether a differently designed assessment could have generated a test outcome that is more concordant with banks' actual performance in the crisis. As we show in this paper, the accuracy of assessment matters as it affects how banks view and respond to regulation, and this has implications for how regulation must be set.

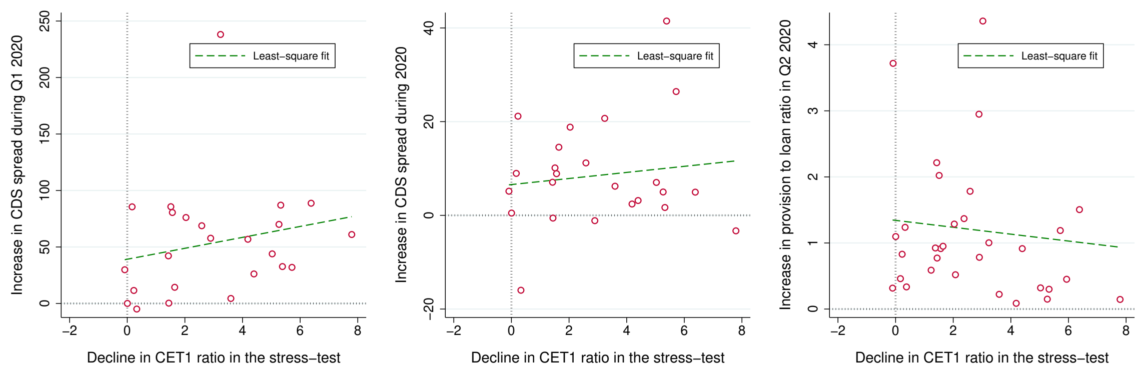

Figure 11: A comparison of the decline in CET1 ratio in the stress-test and the increase in CDS spreads in Q1 2020 (left hand panel), during the full year 2020 (center panel), and loan loss provisions to loans ratio in Q2 2020 (right-hand panel). CDS data is not available for all banks in the sample.

6 Conclusion

Use of supervisory assessments, such as stress-tests, to determine bank-specific regulation has become an important tool for policymakers over time. They have helped regulators in gauging banks' idiosyncratic risks and in bolstering financial stability. Assessments continue to evolve and improve based on lessons learnt over the years. Despite these enhancements, assessments continue to be noisy, not least due to fundamental difficulties inherent in identifying risks. Given that they underpin banks' capital requirements, noisy assessments can lead to misdirected requirements and can have a large impact on banks' capital costs, on their operational incentives, and on overall economic welfare.

To assess the implications, we build a model of supervisory assessment via stress-tests, and show that noisy assessments not only reduce welfare directly, but also by creating adverse ex-ante incentives. Going against the conventional wisdom, we show that in the presence of information frictions, higher capital requirements may lead to more risky banks. Because of this moral hazard issue, supervisory assessment should be used to determine regulatory actions to an even lesser extent when assessment accuracy is lower.

The parsimony and tractability of our model makes it amenable to extensions of interest. For instance, some regulators have discussed maintaining a surprise element in stress-tests on the grounds that it can help avoid pre-positioning or complacency by banks.[41] The welfare effects of surprise in stress tests is not obvious because while it can limit the scope for gaming by banks, higher regulatory uncertainty can weaken the link between the effort banks exert and their performance in the stress-test. This can make banks exert less effort towards improving their risk-return profile. Our model can be extended to study this trade-off by allowing for uncertainty around type-specific capital requirements.

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[41]See, for instance, the remarks by Mr Jerome H Powell, Chair of US Federal Reserve System, at the research conference titled "Stress Testing: A Discussion and Review" on 9 July 2019. In fact, the continuous evolution of the stress-test regime may be motivated by this pursuit.

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Appendix

A Proof of Lemma 2

Assume that the inefficiency term is positive. Then, $\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi$ must also be positive (since integral is increasing in ψ). But this is a contradiction since the overall expression for the planner's FOC must equal zero (or both terms must be zero, which is trivial). As such, the inefficiency term must be negative. In turn, this implies that $\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi > 0$. We know that d^{CE} (the level of deposits in the competitive equilibrium) satisfies $\beta \int_{\frac{Rd}{g(k+d)}}^{\infty} \Big(\psi g'(k+d) - R\Big) f_s(\psi) d\psi = 0$. But since $g(.)$ is concave, it must be that $d^{CE} > d^*$ where d^* solves the constrained planner's problem.

B Endogenous accuracy: Case I

The regulator's problem in this case is as follows:

$$\max_{x,y} \quad \beta p\Big(q_H(y)U_H(\mathcal{X}^o) + (1-q_H(y))U_H(\mathcal{X}^o + x)\Big) + \beta(1-p)\Big(q_L U_L(\mathcal{X}^o) + (1-q_L)U_L(\mathcal{X}^o + x)\Big) - \gamma_c y$$

The first order conditions are:

$$[x] \quad 0 = p(1-q_H(y))U_H'(\mathcal{X}^o + x) + (1-p)(1-q_L)U_L'(\mathcal{X}^o + x)$$

$$[y] \quad 0 = \beta p q_H'(y)(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x)) - \gamma_c$$

Next consider an increase in the cost of accuracy γ_c. A total derivative of the FOCs leads to:

$$[x]: \quad 0 = p\Big((1-q_H(y))U_H''(\mathcal{X}^o + x)\dot{x} - q_H'(y)\dot{y}U_H'(\mathcal{X}^o + x)\Big) +$$
$$(1-p)\Big((1-q_L)U_L''(\mathcal{X}^o + x)\dot{x}\Big)$$

$$[y]: \quad 0 = \beta p\Big(q''_H(y)\dot{y}(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x)) - q'_H(y)U'_H(\mathcal{X}^o + x)\dot{x}\Big) - 1$$

where $\dot{y} = \frac{\partial y}{\partial \gamma_c}$ and $\dot{x} = \frac{\partial x}{\partial \gamma_c}$. The first total derivative implies that $\dot{x}$ and $\dot{y}$ are of the same sign since U is concave, $U'_H < 0$, and $q'_H > 0$. This means that accuracy and surcharge go hand in hand. To assess the direction of change, consider the second total derivative, and replace $\dot{x}$ using the first total derivative. This results in the following expression:

$$\dot{y}\Big(\beta p q''_H(y)(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x)) - \frac{\beta(p q'_H(y)U'_H(\mathcal{X}^o + x))^2}{p(1 - q_H(y))U''_H(\mathcal{X}^o + x) + (1 - p)(1 - q_L)U''_L(\mathcal{X}^o + x)}\Big) = 1$$

The coefficient on $\dot{y}$ is negative following the second-order sufficiency optimality condition (i.e. negative-definite Hessian matrix). This implies that $\dot{y} < 0$, and from the above discussion, that $\dot{x} < 0$.

C Endogenous accuracy: Case II

The regulator's problem in this case is as follows, where both q_H and q_L increase as y, the underlying accuracy of the test, increases:

$$\max_{x,y} \quad \beta p\Big(q_H(y)U_H(\mathcal{X}^o) + (1 - q_H(y))U_H(\mathcal{X}^o + x)\Big) +$$

$$\beta(1 - p)\Big(q_L(y)U_L(\mathcal{X}^o) + (1 - q_L(y))U_L(\mathcal{X}^o + x)\Big) - \gamma_c y$$

The first order conditions are:

$$[x] \quad 0 = p(1 - q_H(y))U'_H(\mathcal{X}^o + x) + (1 - p)(1 - q_L(y))U'_L(\mathcal{X}^o + x)$$

$$[y] \quad 0 = \beta p q'_H(y)(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o + x)) + \beta(1 - p)q'_L(y)(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o + x)) - \gamma_c$$

Next consider an increase in the cost of accuracy γ_c. A total derivative of the FOCs leads to:

$$[x]: \quad 0 = p\Big((1-q_H(y))U_H''(\mathcal{X}^o+x)\dot{x} - q_H'(y)\dot{y}U_H'(\mathcal{X}^o+x)\Big)+$$

$$(1-p)\Big((1-q_L(y))U_L''(\mathcal{X}^o+x)\dot{x} - q_L'(y)\dot{y}U_L'(\mathcal{X}^o+x)\Big)$$

$$[y]: \quad 0 = \beta p\Big(q_H''(y)\dot{y}(U_H(\mathcal{X}^o) - U_H(\mathcal{X}^o+x)) - q_H'(y)U_H'(\mathcal{X}^o+x)\dot{x}\Big)+$$

$$\beta(1-p)\Big(q_L''(y)\dot{y}(U_L(\mathcal{X}^o) - U_L(\mathcal{X}^o+x)) - q_L'(y)U_L'(\mathcal{X}^o+x)\dot{x}\Big) - 1$$

where $\dot{y} = \frac{\partial y}{\partial \gamma_c}$ and $\dot{x} = \frac{\partial x}{\partial \gamma_c}$. The first total derivative no longer (compare to Proposition 4) implies that $\dot{x}$ and $\dot{y}$ are of the same sign given that U is concave, $U_H' < 0$, $q_H' > 0$, $U_L' > 0$, and $q_L' > 0$. This means that accuracy and surcharge do not go hand in hand. To assess the direction of change in y, consider the second total derivative, and replace $\dot{x}$ using the first total derivative. Like before, the second-order sufficiency condition for optimality (ie negative-definite Hessian matrix) implies that $\dot{y} < 0$.

D Optimal surcharge with failure costs

The regulator's problem is as follows:

$$\max_x \quad W(x) = \beta p\Big(q_H U_H(\mathcal{X}^o, \Delta) + (1-q_H)U_H(\mathcal{X}^o + x, \Delta)\Big)$$

$$\beta(1-p)\Big(q_L U_L(\mathcal{X}^o, \Delta) + (1-q_L)U_L(\mathcal{X}^o + x, \Delta)\Big)$$

Here Δ in the utility function formally expresses the dependence of welfare on failure costs. The attendant first-order condition is as follows, where the D_i operator indicates the derivative with respect to the i^{th} argument of U:

$$p(1-q_H)D_1U_H(\mathcal{X}^o + x, \Delta) + (1-p)(1-q_L)D_1U_L(\mathcal{X}^o + x, \Delta) = 0$$

Next, we take the total derivative of this expression with respect to Δ:

$$p(1-q_H)\Big(D_{11}U_H(\chi^o+x,\Delta)\frac{dx}{d\Delta}+D_{12}U_H(\chi^o+x,\Delta)\Big)+$$

$$(1-p)(1-q_L)\Big(D_{11}U_L(\chi^o+x,\Delta)\frac{dx}{d\Delta}+D_{12}U_L(\chi^o+x,\Delta)\Big)=0$$

$$\implies -\underbrace{\Big(p(1-q_H)D_{11}U_H(\chi^o+x,\Delta)+(1-p)(1-q_L)D_{11}U_L(\chi^o+x,\Delta)\Big)}_{A}\frac{dx}{d\Delta}=$$

$$p(1-q_H)D_{12}U_H(\chi^o+x,\Delta)+(1-p)(1-q_L)D_{12}U_L(\chi^o+x,\Delta)$$

Since both U_H and U_L are concave functions of x, $A<0$. To sign the RHS, consider $U_s, s=\{H,L\}$:

$$U_s(\chi^o+x,\Delta)=\bar{Y}-d+\beta g(k+d)(\mu_s-\Delta\int_0^{\frac{Rd}{g(k+d)}}\psi f_s(\psi)d\psi)\quad where\quad d=\frac{k}{\chi^o+x}$$

$$\implies D_2U_s(\chi^o+x,\Delta)=-\beta g(k+d)\int_0^{\frac{Rd}{g(k+d)}}\psi f_s(\psi)d\psi)$$

$$\implies D_{21}U_s(\chi^o+x,\Delta)=-\beta\Bigg(g'(k+d)\frac{dd}{dx}\int_0^{\frac{Rd}{g(k+d)}}\psi f_s(\psi)d\psi+$$

$$g(k+d)\frac{d}{dd}\left[\frac{Rd}{g(k+d)}\right]\frac{Rd}{g(k+d)}f_s\left(\frac{Rd}{g(k+d)}\right)\frac{dd}{dx}\Bigg)$$

As x increases, d decreases i.e. $\frac{dd}{dx}<0$. Also, as d increases, the upper limit on the integral is increases (recall $g(.)$ is concave), which means that by application of Leibniz rule, $D_{21}U_s(\chi^o+x,\Delta)>0$. Since U is a continuous function in both its arguments, $D_{21}U_s(\chi^o+x,\Delta)=D_{12}U_s(\chi^o+x,\Delta)>0$ for both $s=H,L$.